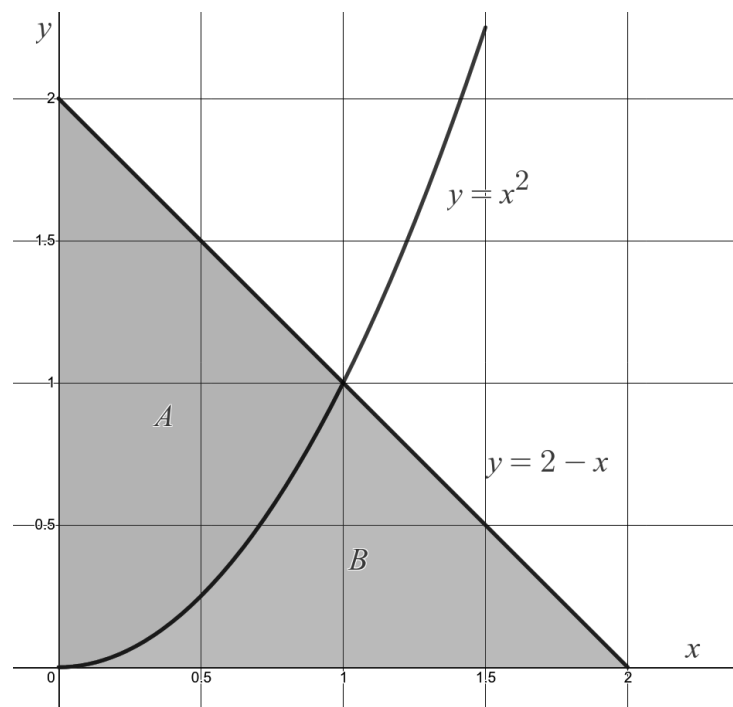**Question 1:** For this question use the following figure:

(i) Determine the volume of the solid produced when region A is rotated about the x-axis.

[5]

(ii) Write down BUT DO NOT EVALUATE a definite integral for the volume of the solid generated when region A is rotated about the line $y = -1$.

[3]

(iii) Write down BUT DO NOT EVALUATE a definite integral for the volume of the solid generated when region B is rotated about the y-axis.

[2]

Question 2: A pyramid has height h and a square base of side length b. The pyramid is divided into three horizontal levels (or floors) each of height $h/3$. <u>Use integration</u> to determine the volume of the middle level.


[5]
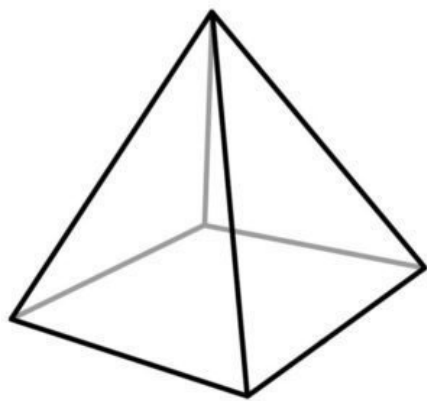
Question 3: Recall that the graph of $y = \sqrt{1 - x^2}$ is the top half of a circle of radius 1 and center $(0, 0)$. Use integration (the arc length formula) to find the length of this curve. (We know what the answer should be; use integration to show that you get the correct result.)

[5]

Question 4: A building elevator system has car of mass 500 kg and a steel cable of linear density 2 kg/m attached to it. When the elevator car is called to the top floor the electric motor pulling the cable shortens it from 20 m to 0 m. How much work did the motor do? Recall that acceleration due to gravity is $g = 9.8$ m/s^2, however you may leave the constant g in your final answer .

[5]

Question 5: The line $y = x$, $0 \leq x \leq 1$, is rotated about the y-axis to form a cone-shaped vessel which is then filled with water to a depth of $1/2$ m. (Here the units for x and y are in meters.) Find the work required to empty the vessel by pumping all of the water to the top of the tank. Recall the density of water is $\rho = 1000$ kg/m^3 and acceleration due to gravity is $g = 9.8$ m/s^2, however you may leave the constants ρ and g in your final answer.
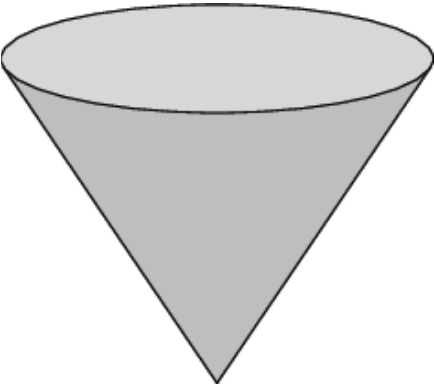
[5]

Question 6: Solve the differential equation with given initial condition. State your final answer in explicit form (that is, isolate y in your final answer.)

$$y' \tan(x) = \sqrt{3} + y, \quad y(\pi/3) = \sqrt{3}$$

[5]

Question 7: Determine the limit of the sequence with terms $a_n = \sqrt{\dfrac{n+1}{9n+1}}$, $n = 1, 2, 3, \ldots$

[3]

Question 8: Write out the first three terms of the geometric series $\displaystyle\sum_{n=0}^{\infty} \dfrac{\pi^n}{5^{n+1}}$ and then decide if it converges. If it does converge then state the sum.

[4]

Question 9: Does the series $\displaystyle\sum_{n=0}^{\infty} \dfrac{e^n}{n^2}$ converge? Explain.

[3]