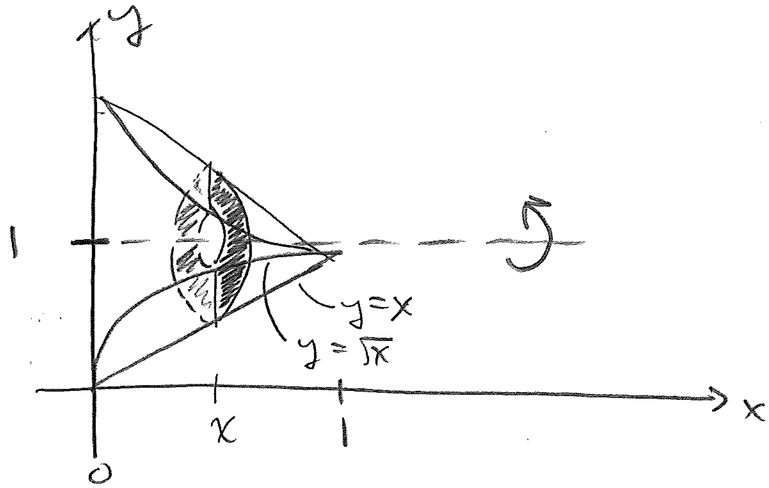
(1) Determine the volume of the solid obtained by rotating the region bounded by $y = x$ and $y = \sqrt{x}$ about the line $y = 1$.



$$V = \int_{x=0}^{1} \left[\pi (1-x)^2 - \pi (1-\sqrt{x})^2\right] dx$$

$$= \pi \int_0^1 \left[\cancel{1} - 2x + x^2 - \cancel{1} + 2x^{1/2} - x\right] dx$$

$$= \pi \int_0^1 \left(x^2 - 3x + 2x^{\frac{1}{2}}\right) dx$$

$$= \pi \left[\frac{x^3}{3} - \frac{3x^2}{2} + 2\left(\frac{2}{3}\right)x^{3/2}\right]_0^1$$

$$= \pi \left[\frac{1}{3} - \frac{3}{2} + \frac{4}{3}\right]$$

$$= \pi \left[\frac{2 - 9 + 8}{6}\right]$$

$$= \boxed{\frac{\pi}{6}}$$

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(2) Determine the length of the curve $y = \ln(1 - x^2)$, $0 \le x \le \frac{1}{2}$.

$$y' = \frac{-2x}{1-x^2}$$

$$L = \int_0^{\frac{1}{2}} \sqrt{1 + (y')^2}\, dx$$

$$= \int_0^{\frac{1}{2}} \sqrt{1 + \left(\frac{-2x}{1-x^2}\right)^2}\, dx$$

$$= \int_0^{\frac{1}{2}} \sqrt{1 + \frac{4x^2}{(1-x^2)^2}}\, dx$$

$$= \int_0^{\frac{1}{2}} \sqrt{\frac{(1-x^2)^2 + 4x^2}{(1-x^2)^2}}\, dx$$

$$= \int_0^{\frac{1}{2}} \sqrt{\frac{x^4 + 2x^2 + 1}{(1-x^2)^2}}\, dx$$

$$= \int_0^{\frac{1}{2}} \sqrt{\frac{(x^2+1)^2}{(1-x^2)^2}}\, dx$$

$$= \int_0^{\frac{1}{2}} \left(\frac{x^2+1}{1-x^2}\right) dx$$

$$\begin{array}{r} -1 \\ -x^2+1 \,\big)\, \overline{x^2+1} \\ -(x^2-1) \\ \hline 2 \end{array}$$

$$\therefore L = \int_0^{\frac{1}{2}} -1 + \frac{2}{1-x^2}\, dx$$

$$= \int_0^{\frac{1}{2}} -1 + \frac{1}{1-x} + \frac{1}{1+x}\, dx$$

$$= \Big[-x\Big]_0^{\frac{1}{2}} - \Big[\ln|1-x|\Big]_0^{\frac{1}{2}} + \Big[\ln|1+x|\Big]_0^{\frac{1}{2}}$$

$$= -\frac{1}{2} - \ln\left(\frac{1}{2}\right) + \cancel{\ln(1)} + \ln\left(\frac{3}{2}\right) - \cancel{\ln(1)}$$

$$= \ln\left(\frac{3/2}{1/2}\right) - \frac{1}{2}$$

$$= \boxed{\ln(3) - \frac{1}{2}}$$

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