(1) Determine $\int x^2 \sin(x)\,dx = I$

Let $u = x^2$ $dv = \sin(x)\,dx$

$du = 2x\,dx$ $v = -\cos(x)$

$I = \int u\,dv$

$= uv - \int v\,du$

$= -x^2\cos(x) - \int -\cos(x) \cdot 2x\,dx$

$= -x^2\cos(x) + 2\int x\cos(x)\,dx$

$u = x$ $dv = \cos(x)\,dx$

$du = 1\,dx$ $v = \sin x$

$= -x^2\cos(x) + 2\left[uv - \int v\,du\right]$

$= -x^2\cos(x) + 2\left[x\sin(x) - \int \sin(x)\,dx\right]$

$= \boxed{-x^2\cos(x) + 2x\sin(x) + 2\cos(x) + C}$

[5]

(2) Determine $\int \frac{5}{(x+1)(x^2+4)}\,dx = I$

$$\frac{0x^2+0x+5}{(x+1)(x^2+4)} \equiv \frac{A}{x+1} + \frac{Bx+C}{x^2+4}$$

$$= \frac{Ax^2+4A+Bx^2+Bx+Cx+C}{(x+1)(x^2+4)}$$

$$= \frac{(A+B)x^2+(B+C)x+(4A+C)}{(x+1)(x^2+4)}$$

① $A+B=0$
② $B+C=0$
③ $4A+C=5$

① $\Rightarrow A=-B$
② $\Rightarrow C=-B$
③ $\Rightarrow 4(-B)+(-B)=5 \Rightarrow -5B=5 \Rightarrow \boxed{B=-1}$

$\therefore A=-B=1$
$C=-B=1$

$$\therefore I = \int \frac{1}{x+1} + \frac{-x+1}{x^2+4}\,dx$$

$$= \int \frac{1}{x+1}\,dx - \underbrace{\left(\frac{1}{2}\right)\int \frac{2x}{x^2+4}\,dx}_{u=x^2+4,\ du=2x\,dx} + \int \frac{1}{x^2+2^2}\,dx$$

$$= \boxed{\ln|x+1| - \frac{1}{2}\ln|x^2+4| + \frac{1}{2}\arctan\left(\frac{x}{2}\right) + C}$$

[10]