(1) Use the definition of the definite integral in the form

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

to evaluate

$$I = \int_1^2 (6x^2 - x)\,dx$$

Carefully set up the Riemann sum and clearly show the steps of your simplification.

$[a,b] = [1,2]$

$\Delta x = \frac{b-a}{n} = \frac{2-1}{n} = \frac{1}{n}$

$x_i = a + i\Delta x = 1 + \frac{i}{n}$

$f(x_i) = 6x_i^2 - x_i$

$= 6\left(1+\frac{i}{n}\right)^2 - \left(1+\frac{i}{n}\right)$

$= 6\frac{i^2}{n^2} + 11\frac{i}{n} + 5$

$$I = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(6\frac{i^2}{n^2} + 11\frac{i}{n} + 5\right)\left(\frac{1}{n}\right)$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{6i^2}{n^3} + \frac{11i}{n^2} + \frac{5}{n}\right)$$

$$= \lim_{n\to\infty} \left[\frac{6}{n^3}\sum_{i=1}^{n} i^2 + \frac{11}{n^2}\sum_{i=1}^{n} i + \frac{1}{n}\sum_{i=1}^{n} 5\right]$$

$$= \lim_{n\to\infty} \left[\frac{\cancel{6}}{n^3} \cdot \frac{n(n+1)(2n+1)}{\cancel{6}} + \frac{11}{n^2}\,\frac{n(n+1)}{2} + \frac{1}{\cancel{n}}\cdot 5\cancel{n}\right]$$

$$= \lim_{n\to\infty} \left[\frac{n}{n}\cdot\frac{n+1}{n}\cdot\frac{2n+1}{n} + \frac{11}{2}\cdot\frac{n}{n}\cdot\frac{n+1}{n} + 5\right]$$

$$= 2 + \frac{11}{2} + 5$$

$$= \boxed{\frac{25}{2}}$$

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(2) Check your answer to question (1) by using the evaluation theorem to compute $\int_1^2 (6x^2 - x)\,dx = I$

$$I = \left[\frac{\cancel{6}^{2} x^3}{\cancel{3}} - \frac{x^2}{2}\right]_1^2$$

$$= \left(2(2^3) - \frac{2^2}{2}\right) - \left(2(1^3) - \frac{1^2}{2}\right)$$

$$= 16 - 2 - 2 + \frac{1}{2}$$

$$= \boxed{\frac{25}{2}}$$

[3]