

1 General Limit Laws

Suppose $\lim_{x \rightarrow a} f(x)$ and $\lim_{x \rightarrow a} g(x)$ both exist, and let c be any constant.

1. Sum Law:
$$\lim_{x \rightarrow a} [f(x) + g(x)] = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x)$$

2. Difference Law:
$$\lim_{x \rightarrow a} [f(x) - g(x)] = \lim_{x \rightarrow a} f(x) - \lim_{x \rightarrow a} g(x)$$

3. Constant Multiplier Law:
$$\lim_{x \rightarrow a} [cf(x)] = c \lim_{x \rightarrow a} f(x)$$

4. Product Law:
$$\lim_{x \rightarrow a} [f(x)g(x)] = \left(\lim_{x \rightarrow a} f(x) \right) \left(\lim_{x \rightarrow a} g(x) \right)$$

5. Quotient Law:
$$\lim_{x \rightarrow a} \left[\frac{f(x)}{g(x)} \right] = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)} \text{ provided } \lim_{x \rightarrow a} g(x) \neq 0 .$$

6. Power Law:
$$\lim_{x \rightarrow a} [f(x)]^n = \left[\lim_{x \rightarrow a} f(x) \right]^n \text{ where } n \text{ is a positive integer.}$$

7. Root Law:
$$\lim_{x \rightarrow a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \rightarrow a} f(x)} \text{ where } n \text{ is a positive integer, and where } \lim_{x \rightarrow a} f(x) > 0 \text{ if } n \text{ is even.}$$

2 Particular Limit Results

1. Constants:
$$\lim_{x \rightarrow a} c = c$$

2. Limit of $f(x) = x$:
$$\lim_{x \rightarrow a} x = a$$

3. Polynomials: If $f(x)$ is a polynomial (for eg. $f(x) = 5x^3 - \pi x^2 - 1/2$) then $\lim_{x \rightarrow a} f(x) = f(a)$.

4. Sine & Cosine:
$$\lim_{x \rightarrow a} \sin(x) = \sin(a), \quad \lim_{x \rightarrow a} \cos(x) = \cos(a) .$$

5. Rational Functions: If $f(x)$ and $g(x)$ are polynomials and $g(a) \neq 0$ then $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{f(a)}{g(a)}$.

6. Squeeze Theorem: If $f(x) \leq g(x) \leq h(x)$ for x near a and $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} h(x) = L$, then $\lim_{x \rightarrow a} g(x) = L$.

7. Important Trig Limit:
$$\lim_{x \rightarrow 0} \frac{\sin(x)}{x} = 1$$