Question 1:

(a)[3 points] Let $f(x) = \sinh\left(\cosh^{-1} x\right)$. Find $f'(x)$.

(b)[3 points] Find $f'(1)$ if $f(x) = \arctan\left(\sqrt{x}\right)$.

(c)[4 points] Evaluate $\lim_{x\to\infty} x^2 \sin\left(\frac{1}{x^2}\right)$.

Question 2:

(a)[3 points] Find $f(x)$ if $f'(x) = e^{2x} - \dfrac{1}{\sqrt{x}}$ and $f(0) = 1$.

(b)[4 points] An object initially $s(0) = 2$ m above the surface of the moon is projected vertically upward with an initial velocity of $v(0) = 10$ m/s. Using the fact that acceleration due to gravity on the moon is $a(t) = -1.6$ m/s^2, derive the formula for $s(t)$, the height of the object above the moon's surface at time t seconds.

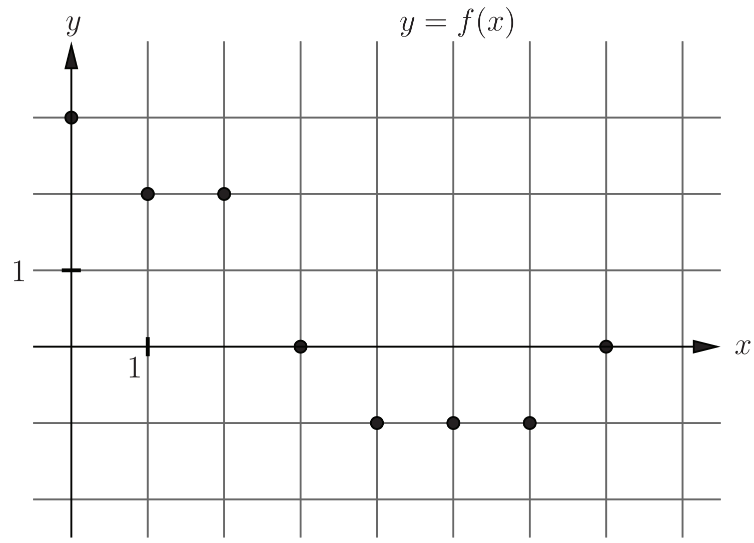
(c)[3 points] Suppose $f(x)$ is a continuous function with the property that

$$\int_0^x f(t)\,dt = \sin(2x) - \int_0^x \cos(2t) f(t)\,dt\ .$$

Find a formula for $f(x)$. (Hint: differentiate both sides of the equation above.)

Question 3:

(a)[5 points] The following figure shows points on the graph of $y = f(x)$. Use the trapezoid rule to estimate $\int_0^7 f(x)\,dx$:



(b)[5 points] $f(x) = xe^x$ has second derivative $f''(x) = (2+x)e^x$. If the midpoint rule is being used to approximate $\int_0^2 xe^x\,dx$, how many subintervals are required in order to be accurate to within 0.01? (Recall, the error in using the midpoint rule to approximate $\int_a^b f(x)\,dx$ is at most $\frac{K(b-a)^3}{24n^2}$, where $|f''(x)| \le K$ on $[a,b]$.)

Question 4:

(a)[5 points] Evaluate $\int 3x^2 - x\sin(x^2)\,dx$.

(b)[5 points] Evaluate $\int_1^e x^{\frac{3}{2}} \ln x \, dx$.

Question 5:

(a)[5 points] Evaluate $\int \tan^2 x \sec^4 x \, dx$.

(b)[5 points] Evaluate $\int \frac{\sqrt{x^2-1}}{x^3} \, dx$. (The identity $\sin(2\theta) = 2\sin\theta\cos\theta$ may be useful here.)

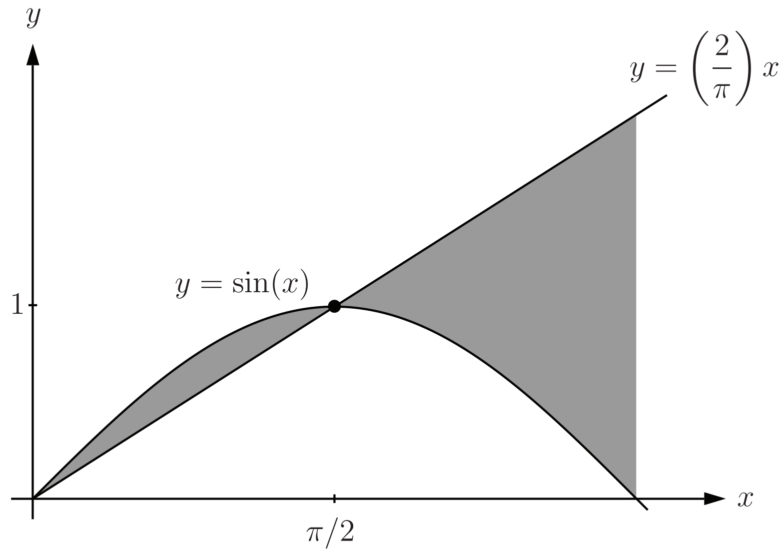
Question 6:

(a)[5 points] Evaluate $\displaystyle\int \frac{1}{x^3+4x^2}\,dx$.

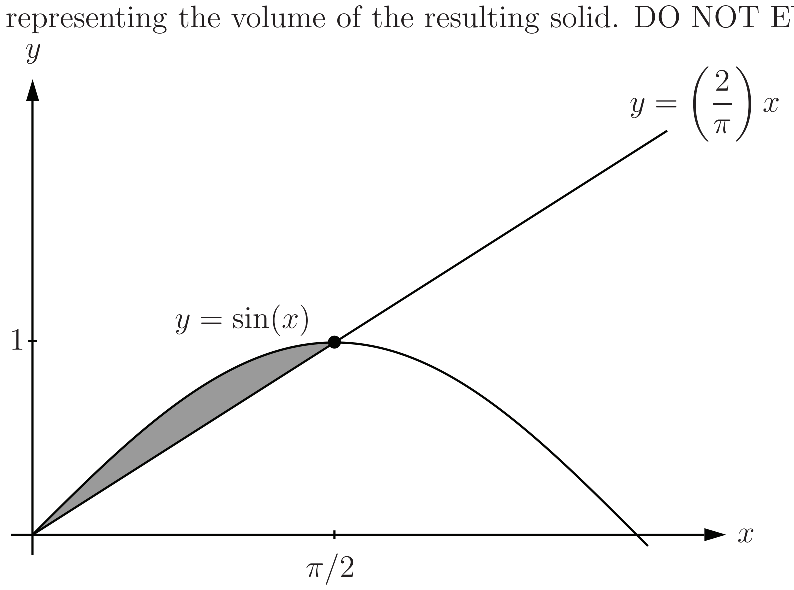
(b)[5 points] Evaluate the improper integral $\displaystyle\int_0^3 \frac{x}{\sqrt{9-x^2}}\,dx$

Question 7:

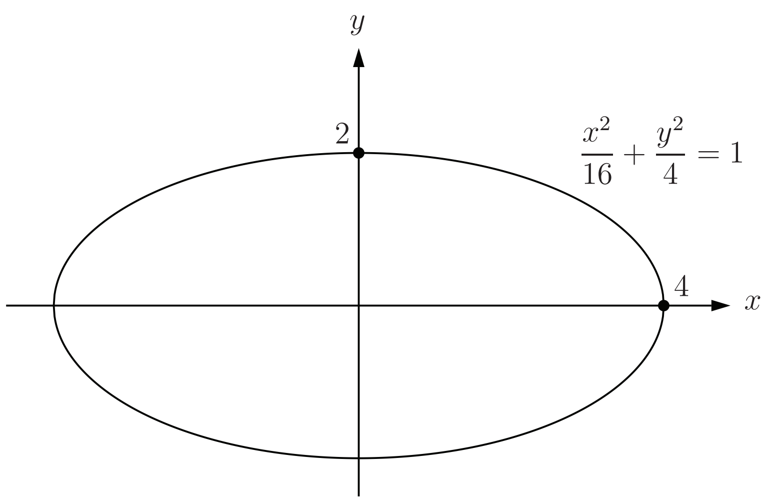
(a)[5 points] Find the area of the shaded region:



(b)[5 points] The shaded region is rotated about the vertical line $x = \pi/2$; set up the integral representing the volume of the resulting solid. DO NOT EVALUATE THE INTEGRAL.



Question 8: Consider the graph of the following ellipse:

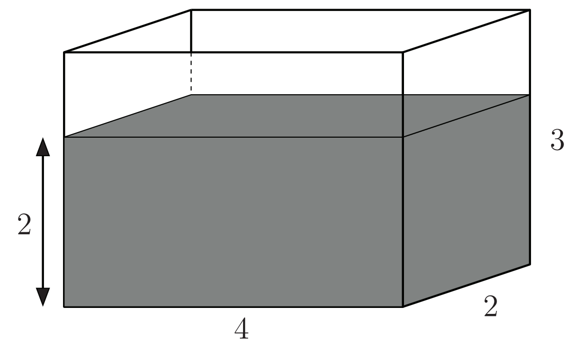


If the ellipse is rotated about the x-axis the resulting solid is called an ellipsoid (which looks rather like a watermelon).

(a)[**3 points**] Isolate y in the equation above to find a function which describes the top half of the ellipse.

(b)[**7 points**] Use your result in (a) to find the volume of the ellipsoid.

Question 9: A rectangular fish tank of length 4 m, width 2 m and height 3 m contains water to a depth of 2 m. Recall that the density of water is $\rho = 1000$ kg/m^3 and acceleration due to gravity is $g = 9.8$ m/s^2.



(a)[5 points] How much work is required to pump all of the water out over the edge of the tank?

(b)[5 points] What is the hydrostatic force (force due to water pressure) exerted on one of the long sides of the tank? Recall that pressure P as a function of depth h is $P(h) = \rho g h$ where ρ is the density of the liquid and g is acceleration due to gravity.

Question 10: The fish population in a large lake is infected by a disease at time $t = 0$, and the declining fish population is described by the differential equation

$$\frac{dP}{dt} = -k\sqrt{P}\ .$$

Here k is a positive constant and $P(t)$ is the fish population at time t weeks. Suppose there were initially 90,000 fish in the lake and that 40,000 remain after 6 weeks.

(a)[**7 points**] Solve the differential equation to find a formula for $P(t)$.

(b)[**3 points**] Use your result in (a) to find the time required for the fish population to reduce to 10,000.

Question 11: Recall that $\sinh(x) = \dfrac{e^x - e^{-x}}{2}$, and that the Maclaurin series for e^x is

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

(a)[**4 points**] Find the first three non-zero terms of the Maclaurin series for $f(x) = \sinh(x^2)$.

(b)[**3 points**] Use a Maclaurin series to evaluate the limit

$$\lim_{x \to 0} \frac{e^x - 1 - x - (x^2/2)}{x^3}$$

(c)[**3 points**] Suppose $f(x)$ is a function such that $f(2) = 3$, $f'(2) = 0$, $f''(2) = -1$ and $f'''(2) = 2$. Use a Taylor polynomial of degree 3 to approximate $f(2.1)$. Round your final answer to three decimals.