Question 1:

(a)[3 points] Evaluate $f'(0)$ where $f(x) = x \arccos(x) - \sqrt{1 - x^2}$.

(b)[3 points] Evaluate $g'(0)$ where $g(x) = \cosh(x) \sinh(x^2)$.

(c)[4 points] Evaluate

$$\lim_{x \to 0^+} \sin(x) \ln(x)$$

Question 2:

(a)[4 points] Evaluate

$$\lim_{t\to 0}\left(\frac{1}{t}-\frac{1}{te^t}\right)$$

(b)[3 points] An animal gains mass at a rate of $W(t)=\dfrac{100t}{(t^2+1)^3}$ kg/yr, where t is time in years. What is the total mass gain during the first two years of life?

(c)[3 points] The average value of $f(x)=qx^2$ over the interval $[-q,q]$ is 9. Determine the value of the constant q.

Question 3:

(a)[3 points] Define the function

$$F(x) = \int_{-1}^{x^3} \frac{\cos(t^2)}{e^t}\, dt$$

Evaluate and simplify $F(-1) - F'(0)$.

(b)[3 points] Evaluate:

$$\int \frac{1}{x \ln x}\, dx$$

(c)[4 points] Evaluate:

$$\int_0^1 (\sqrt[4]{x} + 1)^2\, dx$$

Question 4:

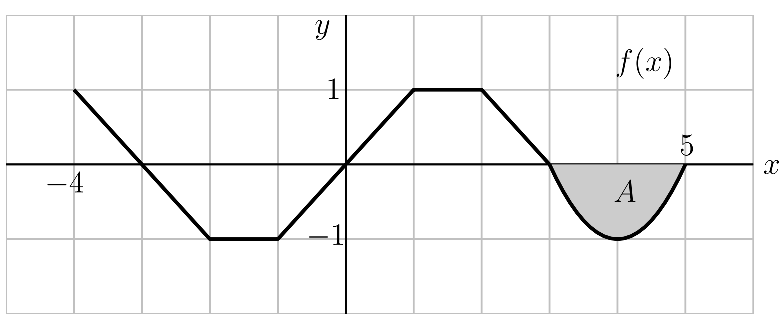
(a)[3 points] Evaluate:

$$\int \frac{x+2}{(x-1)^{3/2}}\, dx$$

(b)[4 points] Evaluate the following limit of Riemann Sums by interpreting it as $\int_a^b f(x)\, dx$ for some a, b and $f(x)$:

$$\lim_{n\to\infty} \frac{1}{n}\left[\left(\frac{1}{n}\right)^3 + \left(\frac{2}{n}\right)^3 + \cdots + \left(\frac{n}{n}\right)^3\right]$$

(c)[3 points] For the function $f(x)$ whose graph is shown below, $\displaystyle\int_{-4}^{5} f(x)\, dx = \frac{-5}{6}$. Determine the area of the shaded region A.



Question 5 [8 points]: Evaluate:

$$\int x^3(\ln x)^2\,dx$$

Question 6 [8 points]: Evaluate:

$$\int \frac{5x-8}{x^2+x-12}\,dx$$

Question 7 [8 points]: Evaluate:

$$\int \sqrt{16 - x^2}\, dx$$

Question 8:

(a) [**4 points**] Consider the integral $\int_0^4 4(1+x^2)\,dx$. Let M_2 be the midpoint rule approximation of the integral using two subintervals, and T_2 the trapezoid rule approximation using two subintervals. Determine $T_2 - M_2$.

(b) [**4 points**] Determine the error in T_2, the trapezoid rule approximation used in part (a). Recall, the error in using the trapezoid rule to approximate $\int_a^b f(x)\,dx$ is at most $\dfrac{K(b-a)^3}{12n^2}$, where $|f''(x)| \le K$ on $[a,b]$.

Question 9:

(a)[4 points] Evaluate the improper integral

$$\int_0^\infty \frac{x^3}{e^{(x^4)}}\,dx$$

(b)[4 points] Evaluate the improper integral:

$$\int_2^3 \frac{1}{\sqrt{3-x}}\,dx$$

(c)[2 points] Use the Comparison Theorem to show that the following integral diverges:

$$\int_1^\infty \frac{x^3+\sin^2 x}{x^4}\,dx$$

Question 10:

(a)[**5 points**] Determine the area of the region bounded between the curves $y = 4x - x^2$ and $y = x^2$.

(b)[**5 points**] The bounded region in part (a) is rotated about the x-axis. Determine the volume of the resulting solid. (Disks/washers would be best here.)

Question 11:

(a)[4 points] The curve $y = \cos(x)$, $-\pi/2 \leq x \leq \pi/2$, is rotated about the vertical line $x = \pi$. Set up but do not evaluate the integral representing the volume of the resulting solid. (Cylinders would be best here.)

(b)[4 points] 20 m rope hangs over the side of a building and a 10 kg bucket is tied to the end of the rope. A person at the top of the building pulls the rope and bucket up onto the roof of the building. How much work is done if the rope has a total mass of 2 kg? Recall that the acceleration due to gravity is $g = 9.8$ m/s^2.

Question 12:

(a)[5 points] The base (flat bottom surface) of a solid is the region between the curve $y = \sqrt{1-x^2}$ and the x-axis. Note that $y = \sqrt{1-x^2}$ is the upper half of the circle of radius 1 and centre $(0,0)$. Cross-sections perpendicular to the x-axis are isosceles triangles of equal height and base. Determine the volume of the solid.

(b)[5 points] Solve the following differential equation:

$$\frac{dy}{dx} = \frac{1+y^2}{\sqrt{x+1}}, \qquad y(0) = 1$$

You may leave your solution in implicit form (it is not necessary to isolate the y variable in your final answer.)

Question 13:

(a)[3 points] Determine the first three non-zero terms of the Maclaurin series for the function $f(x) = x^3 e^{x^2}$.

(b)[4 points] Use a Maclaurin series (not L'Hospital's Rule) to evaluate the limit

$$\lim_{x \to 0} \frac{1 - \cos(x^2)}{x^3(e^x - 1)}$$

(c)[3 points] Determine the Maclaurin polynomial of degree three for $f(x) = e^x \sin x$. You may use the definition or any other valid method to obtain your answer.