(1) Use the definition of the definite integral in the form

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

to evaluate

$$\int_{-1}^{2} \left(x^2 - 3x + 1\right)\,dx$$

Carefully set up the Riemann sum and clearly show the steps of your simplification.

$[a,b] = [-1, 2]$

$\Delta x = \frac{b-a}{n} = \frac{2-(-1)}{n} = \frac{3}{n}$

$x_i = a + i\Delta x = -1 + \frac{3i}{n}$

$f(x_i) = x_i^2 - 3x_i + 1 = \left(-1+\frac{3i}{n}\right)^2 - 3\left(-1+\frac{3i}{n}\right) + 1 = 5 - \frac{15i}{n} + \frac{9i^2}{n^2}$

$$\int_{-1}^{2} (x^2-3x+1)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(5 - \frac{15i}{n} + \frac{9i^2}{n^2}\right)\left(\frac{3}{n}\right)$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{15}{n} - \frac{45}{n^2}i + \frac{27}{n^3}i^2\right)$$

$$= \lim_{n\to\infty} \left[\left(\sum_{i=1}^{n} \frac{15}{n}\right) - \left(\frac{45}{n^2}\right)\left(\sum_{i=1}^{n} i\right) + \left(\frac{27}{n^3}\right)\left(\sum_{i=1}^{n} i^2\right)\right]$$

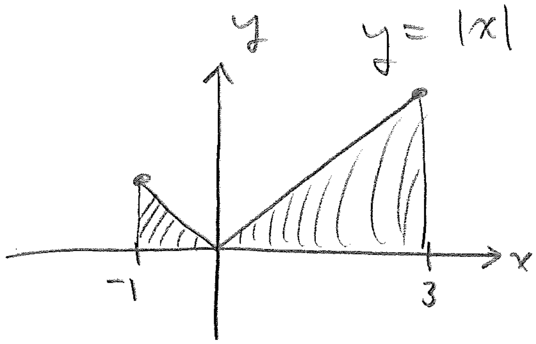
$$= \lim_{n\to\infty} \left[15 \cdot \frac{\cancel{n}}{\cancel{n}} - \frac{45}{2} \cdot \frac{\cancel{n}}{\cancel{n}} \cdot \underbrace{\frac{n+1}{n}}_{\to 1} + \frac{\cancel{27}^{\,9}}{\cancel{6}_{\,2}} \cdot \frac{\cancel{n}}{\cancel{n}} \cdot \underbrace{\frac{n+1}{n}}_{\to 1} \cdot \underbrace{\frac{2n+1}{n}}_{\to 2}\right]$$

$$= 15 - \frac{45}{2} + 9$$

$$= \boxed{\frac{3}{2}}$$

[7]

(2) Compute $\int_{-1}^{3} |x|\, dx$ (hint: area interpretation)



$$\therefore \int_{-1}^{3} |x|\, dx = \left(\tfrac{1}{2}\right)(1)(1) + \left(\tfrac{1}{2}\right)(3)(3)$$

$$= \boxed{5}$$

[4]

(2) Evaluate $\int_{0}^{\pi/4} \frac{1+\cos^2(x)}{\cos^2(x)}\, dx = \int_0^{\frac{\pi}{4}} \sec^2(x) + 1 \; dx$

$$= \Big[\tan(x)\Big]_0^{\frac{\pi}{4}} + \Big[x\Big]_0^{\pi/4}$$

$$= \underbrace{\tan\left(\tfrac{\pi}{4}\right)}_{1} - \underbrace{\tan(0)}_{0} + \frac{\pi}{4} - 0$$

$$= \boxed{1 + \frac{\pi}{4}}$$

or $\boxed{\frac{4+\pi}{4}}$

[4]