(1) For this question let $f(x) = \dfrac{1}{x+2}$

(i) Determine $T_1(x)$, the linear approximation to f at $a = 2$.

$$f(x) = \frac{1}{x+2} \; ; \; f(2) = \frac{1}{2+2} = \frac{1}{4}$$

$$f'(x) = \frac{-1}{(x+2)^2} \; ; \; f'(2) = \frac{-1}{(2+2)^2} = \frac{-1}{16}$$

$$T_1(x) = f(a) + f'(a)(x-a)$$
$$= \boxed{\frac{1}{4} - \frac{1}{16}(x-2)}$$

[3]

(ii) Use your approximation to estimate $f(3/2)$. State your answer as a single simplified fraction.

$$f\left(\tfrac{3}{2}\right) \approx T_1\left(\tfrac{3}{2}\right) = \frac{1}{4} - \frac{1}{16}\left(\frac{3}{2} - 2\right)$$

$$= \frac{1}{4} - \frac{1}{16}\left(\frac{-1}{2}\right)$$

$$= \boxed{\frac{9}{32}}$$

[3]

(iii) Determine an error bound on your approximation in part (ii). Again, state your answer as a single simplified fraction.

$$f'(z) = \frac{-1}{(z+2)^2} \Rightarrow f''(z) = \frac{2}{(z+2)^3}$$

$$R_1(x) = \frac{f''(z)}{2}(x-a)^2 \quad \text{where} \quad x = \frac{3}{2}, \; a = 2, \; \frac{3}{2} < z < 2$$

$$\therefore \left|R_1\left(\tfrac{3}{2}\right)\right| = \left|\left(\frac{1}{2}\right)\left(\frac{2}{[z+2]^3}\right)\left(\frac{3}{2} - 2\right)^2\right|$$

$$\leq \cancel{\left(\frac{1}{2}\right)} \frac{\cancel{2}}{\left(\frac{3}{2}+2\right)^3} \left(\frac{1}{2}\right)^2$$

$$= \left(\frac{2}{7}\right)^3 \left(\frac{1}{2}\right)^2 = \boxed{\frac{2}{7^3}}$$

[4]

(2) Let $f(x) = x^2 \ln(x)$. Find $T_2(x)$, the Taylor polynomial of degree 2 about $a = 1$. Simplify any fractions in your final answer.

$$f(x) = x^2 \ln(x) \quad ; \quad f(1) = 1^2 \ln(1) = 0$$

$$f'(x) = 2x \ln(x) + x^2 \cdot \frac{1}{x} \quad ; \quad f'(1) = (2)(1)\cancel{\ln(1)}^{\,0} + 1 = 1$$

$$= 2x \ln(x) + x$$

$$f''(x) = 2\ln(x) + 2\cancel{x}\left(\frac{1}{\cancel{x}}\right) + 1 \quad ; \quad f''(1) = 2\cancel{\ln(1)}^{\,0} + 3 = 3$$

$$= 2\ln(x) + 3$$

$$\therefore \; T_2(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2$$

$$= 0 + 1(x-1) + \frac{3}{2}(x-1)^2$$

$$\boxed{\therefore \; T_2(x) = (x-1) + \frac{3}{2}(x-1)^2}$$

[5]