Question 1:

(a) Use a linear approximation $T_1(x)$ to estimate $f(-1/6)$ where $f(x) = (2 + e^x)^2$. Simplify your final answer.

[5]

(b) Give an error bound on your approximation in part (a). Again, simplify your final answer.

[5]

Question 2:

(a) Use a Taylor polynomial of degree 2 about $a = 4$ to estimate $\sqrt{5}$. Simplify your final answer.

[5]

(b) Give an error bound on your approximation in part (a). Again, simplify your final answer.

[5]

Question 3:

(a) Find the first three nonzero terms of the Maclaurin series for $f(x) = \dfrac{\arctan(2x)}{1+x^2}$.

[5]

(b) Find the first four nonzero terms of the Taylor series about $a = 2$ for $g(x) = \dfrac{2}{7-3x}$ and state the open interval of convergence.

[5]

Question 4: The first three terms of the Maclaurin series for $\tan(x)$ is

$$\tan(x) = x + \frac{x^3}{3} + \frac{2x^5}{15} + \cdots$$

Use this to find the first three nonzero terms of the Maclaurin series for $g(x) = x^3 \sec^2(x)$. (There are several ways to do this, but one way is much easier than the others.)

[4]

Question 5: Evaluate the following limit:

$$\lim_{x \to 0} \frac{\ln(1 - x^2) - e^{(-x^2)} + 1}{3x^4}$$

[6]

Question 6: Find the radius of convergence R and open interval of convergence $\mathcal{I}$ for the power series

$$f(x) = \sum_{k=0}^{\infty} \frac{(-1)^k (x+3)^{2k}}{9^k}$$

[5]

Question 7: Find the radius of convergence R and open interval of convergence $\mathcal{I}$ for the power series

$$f(x) = \sum_{k=0}^{\infty} \frac{2^k \sqrt{k}\ x^k}{k!}$$

[5]