

(1) Determine $\int x \cos(5x) dx = I$

$$u = x \quad dv = \cos(5x) dx$$

$$du = dx \quad v = \frac{\sin(5x)}{5}$$

$$I = \int u dv = uv - \int v du$$

$$= \frac{x \sin(5x)}{5} - \int \frac{\sin(5x)}{5} dx$$

$$= \boxed{\frac{x \sin(5x)}{5} + \frac{\cos(5x)}{25} + C}$$

(2) Determine either $\int \frac{1}{(x+5)^2(x-1)} dx$ or $\int \frac{x^2+1}{(x-3)(x-2)^2} dx$ (circle the integral you are attempting.)

 I_1 I_2

$$\begin{aligned} \text{For } I_1: \quad \frac{1}{(x+5)^2(x-1)} &= \frac{A}{x+5} + \frac{B}{(x+5)^2} + \frac{C}{x-1} \\ &= \frac{A(x+5)(x-1) + B(x-1) + C(x+5)^2}{(x+5)^2(x-1)} \\ &= \frac{(A+C)x^2 + (4A+B+10C)x + (-5A-B+25C)}{(x+5)^2(x-1)} \end{aligned}$$

$$\therefore A+C=0 \Rightarrow C=-A$$

$$4A+B+10C=0 \Rightarrow B=-4A-10C=-4A-10(-A)=6A$$

$$-5A-B+25C=1 \Rightarrow -5A-(6A)+25(-A)=1$$

$$\therefore -36A=1$$

$$A = -\frac{1}{36}$$

$$\therefore B=6A = -\frac{1}{6}$$

$$\therefore C=-A = \frac{1}{36}$$

$$\begin{aligned} \therefore I_1 &= \int \frac{(-1/36)}{x+5} dx + \int \frac{(-1/6)}{(x+5)^2} dx + \int \frac{(1/36)}{x-1} dx \\ &= \boxed{-\frac{1}{36} \ln|x+5| + \frac{1}{6} \cdot \frac{1}{(x+5)} + \frac{1}{36} \ln|x-1| + C} \end{aligned}$$

$$\begin{aligned} \text{For } I_2: \quad \frac{x^2+1}{(x-3)(x-2)^2} &= \frac{A}{x-3} + \frac{B}{x-2} + \frac{C}{(x-2)^2} \\ &= \frac{A(x-2)^2 + B(x-3)(x-2) + C(x-3)}{(x-3)(x-2)^2} \\ &= \frac{(A+B)x^2 + (-4A-5B+C)x + (4A+6B-3C)}{(x-3)(x-2)^2} \end{aligned}$$

$$\therefore A+B=1 \Rightarrow B=1-A$$

$$-4A-5B+C=0 \Rightarrow C=4A+5B=4A+5(1-A)=5-A$$

$$4A+6B-3C=1 \Rightarrow 4A+6(1-A)-3(5-A)=1 \Rightarrow A=10$$

$$\left. \begin{array}{l} \therefore A=10, \\ B=1-10=-9 \\ C=5-10=-5 \end{array} \right\}$$

$$\therefore I_2 = \int \frac{10}{x-3} dx + \int \frac{-9}{x-2} dx + \int \frac{-5}{(x-2)^2} dx$$

$$= \boxed{10 \ln|x-3| - 9 \ln|x-2| + \frac{5}{x-2} + C}$$

[10]