 Name: SOLUTIONS

(i) Use the definition of the definite integral in the form

$$\int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$$

to evaluate

$$\int_1^3 (3x^2 - 2x)\ dx$$

Carefully set up the Riemann sum and clearly show the steps of your simplification.

$[a,b] = [1,3]$

$\Delta x = \frac{b-a}{n} = \frac{3-1}{n} = \frac{2}{n}$

$x_i = a + i\Delta x = 1 + \frac{2i}{n}$

$f(x_i) = 3x_i^2 - 2x_i$

$= 3\left(1+\frac{2i}{n}\right)^2 - 2\left(1+\frac{2i}{n}\right)$

$= 1 + \frac{8i}{n} + \frac{12i^2}{n^2}$

$$\begin{aligned}
&\int_1^3 (3x^2 - 2x)dx \\
&= \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x \\
&= \lim_{n\to\infty}\sum_{i=1}^{n}\left(1+\frac{8i}{n}+\frac{12i^2}{n^2}\right)\left(\frac{2}{n}\right) \\
&= \lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{2}{n}+\frac{16i}{n^2}+\frac{24i^2}{n^3}\right) \\
&= \lim_{n\to\infty}\left[\left(\frac{2}{n}\sum_{i=1}^{n}1\right)+\left(\frac{16}{n^2}\sum_{i=1}^{n}i\right)+\left(\frac{24}{n^3}\sum_{i=1}^{n}i^2\right)\right] \\
&= \lim_{n\to\infty}\left[\frac{2}{\cancel{n}}\cdot\cancel{n} + \frac{16}{n^2}\frac{n(n+1)}{2} + \frac{24}{n^3}\frac{n(n+1)(2n+1)}{6}\right] \\
&= \lim_{n\to\infty}\left[2 + \frac{16}{2}\cdot\frac{\cancel{n}}{\cancel{n}}\cdot\underbrace{\frac{n+1}{n}}_{\to 1} + \frac{24}{6}\cdot\frac{\cancel{n}}{\cancel{n}}\cdot\underbrace{\frac{n+1}{n}}_{\to 1}\cdot\underbrace{\frac{2n+1}{n}}_{\to 2}\right] \\
&= 2 + 8 + 8 \\
&= \boxed{18}
\end{aligned}$$

[8]

(ii) Now calculate $\int_1^3 (3x^2 - 2x)\ dx$ using the Fundamental Theorem of Calculus (Part 2) to check your answer in part (i).

$$\begin{aligned}
\int_1^3 (3x^2 - 2x)dx &= \left[\frac{\cancel{3}x^3}{\cancel{3}} - \frac{\cancel{2}x^2}{\cancel{2}}\right]_1^3 \\
&= (3^3 - 3^2) - \cancel{(1^3 - 1^2)}^{\to 0} \\
&= \boxed{18}
\end{aligned}$$

[2]