Math 122 - Quiz 1 Name: SOLUTIONS.

(1) For this question let $f(x) = \dfrac{1}{\sqrt{1+3x}}$

(i) Determine $T_1(x)$, the linear approximation to f at $a = 0$.

$f(x) = (1+3x)^{-\frac{1}{2}}$; $f(0) = 1$

$f'(x) = -\frac{1}{2}(1+3x)^{-3/2} \cdot 3$; $f'(0) = -\frac{3}{2}$

$\therefore \quad T_1(x) = f(a) + f'(a)(x-a)$

$$\boxed{T_1(x) = 1 - \tfrac{3}{2}x}$$

[3]

(ii) Use your approximation to estimate $f(1/6)$. State your answer as a single simplified fraction.

$f(\frac{1}{6}) \approx T_1(\frac{1}{6}) = 1 - (\frac{\cancel{3}}{2})(\frac{1}{\cancel{6}\,2}) = \boxed{\frac{3}{4}}$

[3]

(iii) Determine an error bound on your approximation in part (ii). Again, state your answer as a single simplified fraction.

$f''(z) = (-\frac{3}{2})(-\frac{3}{2})(1+3z)^{-5/2}(3) = \dfrac{27}{4(1+3z)^{5/2}}$

$\therefore \quad R_1(x) = \dfrac{f''(z)}{2}(x-a)^2 = (\frac{1}{2})\dfrac{27}{4(1+3z)^{5/2}}(x-0)^2$, z between a & x,

$\therefore \quad 0 < z < \frac{1}{6}$.

$\therefore \left|R_1(\tfrac{1}{6})\right| \leq (\frac{1}{2})\left[\dfrac{27}{4(1+3\cdot 0)^{5/2}}\right](\frac{1}{6})^2$

$= \dfrac{3^3}{2^3 \cdot 2^2 \cdot 3^2} = \boxed{\dfrac{3}{32}}$

[4]

(2) Let $f(x) = \cos(2x)$. Find $T_3(x)$, the Taylor polynomial of degree 3 about $a = \pi/4$. Simplify any fractions in your final answer.

$$f(x) = \cos(2x) \quad ; \quad f\left(\tfrac{\pi}{4}\right) = \cos\left(2\cdot\tfrac{\pi}{4}\right) = 0$$

$$f'(x) = -2\sin(2x) \quad ; \quad f'\left(\tfrac{\pi}{4}\right) = -2\sin\left(2\cdot\tfrac{\pi}{4}\right) = -2.$$

$$f''(x) = -4\cos(2x) \quad ; \quad f''\left(\tfrac{\pi}{4}\right) = -4\cos\left(2\cdot\tfrac{\pi}{4}\right) = 0$$

$$f'''(x) = 8\sin(2x) \quad ; \quad f'''\left(\tfrac{\pi}{4}\right) = 8\sin\left(2\cdot\tfrac{\pi}{4}\right) = 8$$

$$T_3(x) = 0 - 2\left(x-\tfrac{\pi}{4}\right) + \frac{0}{2!}\left(x-\tfrac{\pi}{4}\right)^2 + \frac{8}{3!}\left(x-\tfrac{\pi}{4}\right)^3$$

$$\boxed{T_3(x) = -2\left(x-\tfrac{\pi}{4}\right) + \frac{4}{3}\left(x-\tfrac{\pi}{4}\right)^3}$$

[5]