 Name: SOLUTIONS

(1) For this question let $f(x) = \dfrac{1}{\sqrt{1+4x}}$

(i) Determine $T_1(x)$, the linear approximation to f at $a = 0$.

$f(x) = (1+4x)^{-\frac{1}{2}}$; $f(0) = 1$

$f'(x) = -\frac{1}{2}(1+4x)^{-3/2}(4)$; $f'(0) = -2$

$\therefore T_1(x) = f(a) + f'(a)(x-a)$

$$\boxed{T_1(x) = 1 - 2x}$$

[3]

(ii) Use your approximation to estimate $f(1/4)$. State your answer as a single simplified fraction.

$f(\frac{1}{4}) \approx T_1(\frac{1}{4}) = 1 - 2(\frac{1}{4}) = \boxed{\frac{1}{2}}$

[3]

(iii) Determine an error bound on your approximation in part (ii). Again, state your answer as a single simplified fraction.

$f''(z) = (-\cancel{2})(\frac{-3}{\cancel{2}})(1+4z)^{-5/2}(4) = \dfrac{12}{(1+4z)^{5/2}}$

$R_1(x) = \dfrac{f''(z)}{2}(x-a)^2 = (\frac{1}{2})\left(\dfrac{12}{(1+4z)^{5/2}}\right)(x-0)^2$, z between a & x

$\therefore 0 < z < \frac{1}{4}$

$\therefore \left|R_1(\frac{1}{4})\right| \leq (\frac{1}{2})\left(\dfrac{12}{(1+4\cdot 0)^{5/2}}\right)(\frac{1}{4})^2$

$= \dfrac{2^2 \cdot 3}{2^5} = \boxed{\dfrac{3}{8}}$

[4]

(2) Let $f(x) = \sin(2x)$. Find $T_4(x)$, the Taylor polynomial of degree 4 about $a = \pi/4$. Simplify any fractions in your final answer.

$f(x) = \sin(2x)$; $f(\frac{\pi}{4}) = \sin(2\cdot\frac{\pi}{4}) = 1$

$f'(x) = 2\cos(2x)$; $f'(\frac{\pi}{4}) = 2\cos(2\cdot\frac{\pi}{4}) = 0$

$f''(x) = -4\sin(2x)$; $f''(\frac{\pi}{4}) = -4\sin(2\cdot\frac{\pi}{4}) = -4$

$f'''(x) = -8\cos(2x)$; $f'''(\frac{\pi}{4}) = -8\cos(2\cdot\frac{\pi}{4}) = 0$

$f^{(4)}(x) = 16\sin(2x)$; $f^{(4)}(\frac{\pi}{4}) = 16\sin(2\cdot\frac{\pi}{4}) = 16.$

$\therefore T_4(x) = 1 + 0\cdot(x-\frac{\pi}{4}) - \frac{4}{2!}(x-\frac{\pi}{4})^2 + \frac{0}{3!}(x-\frac{\pi}{4})^3 + \frac{16}{4!}(x-\frac{\pi}{4})^4$

$$\boxed{T_4(x) = 1 - 2(x-\tfrac{\pi}{4})^2 + \tfrac{2}{3}(x-\tfrac{\pi}{4})^4}$$

[5]