Question 1: Find the radius of convergence R and open interval of convergence $\mathcal{I}$ for the power series

$$f(x) = \sum_{k=0}^{\infty} \frac{2^{2k}(x+3)^k}{k!}$$

[5]

Question 2: Determine $f(x)$ if

$$f''(x) = 12x^2 - \sin(x), \qquad f(0) = 1, \qquad f'(0) = 0$$

[5]

Question 3: Use the definition of the definite integral in the form

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

to evaluate

$$\int_1^2 (3x^2 - 1)\,dx$$

Carefully set up the Riemann sum and clearly show the steps of your simplification.

[10]

Question 4: The daytime temperature T over a 12 hour period is modelled by the function

$$T(t) = 10 + 6\sin\left(\frac{\pi t}{12}\right)$$

where T is in degrees celsius and t is in hours, and where $t = 0$ corresponds to 6:00 a.m. Determine the average temperature over the 12 hour period from 6:00 a.m. to 6:00 p.m.

[5]

Question 5: Determine the following:

(i) $\displaystyle\int \frac{3x^2 - 2x + 1}{x}\,dx$

[2]

(ii) $\displaystyle\int_0^{\pi/4} \sec^2(x) - \frac{4}{\pi}\,dx$

[3]

Question 6: Determine

$$\int_{-1}^{1} x e^{x^2}\, dx$$

[5]

Question 7: Determine

$$\int \frac{\cos\left(\ln\left(x\right)\right)}{x}\, dx$$

[5]

Question 8: Determine

$$\int_0^1 x^2(1+x^3)^5\,dx$$

[5]

Question 9: Determine

$$\int x\sqrt{1+x}\,dx$$

[5]