Question 1:

(a) Find the linear approximation $T_1(x)$ about $a = 1$ for $f(x) = x^{7/2}$.

[4]

(b) Use your result in part (a) to approximate $(0.9)^{\frac{7}{2}}$. (Express your answer as a single simplified fraction.)

[2]

(c) Give an error bound for your approximation in part (b). (Again, express your answer as a single simplified fraction.)

[4]

Question 2:

(a) Find the Taylor polynomial of degree 3 for $f(x) = \cos(2 + 2x)$ at $a = -1$.

[5]

(b) Suppose $T_3(x)$ in part (a) is used to approximate $f(-3/2)$. Give an error bound on the approximation. Express your answer as a single simplified fraction. (Note: you are not being asked to find the approximation to $f(-3/2)$ here, but only the error bound associated with the approximation.)

[5]

Question 3:

(a) Find the first four nonzero terms of the Maclaurin series for $f(x) = x^3 e^{-x^2}$.

[5]

(b) Determine $f^{(11)}(0)$, the eleventh derivative of f at 0, where again $f(x) = x^3 e^{-x^2}$. There is no need to simplify factorials (if any) in your final answer.

[3]

(c) Determine $f^{(12)}(0)$ where again $f(x) = x^3 e^{-x^2}$. There is no need to simplify factorials (if any) in your final answer.

[2]

Question 4:

(a) Find the first four nonzero terms of the Maclaurin series for $f(x) = \dfrac{3x}{(1-x)^2}$ and state the open interval of convergence.

(Hint: think about the Maclaurin series for the derivative of $\dfrac{1}{1-x}$.)

[5]

(b) Find the the first four nonzero terms of the Taylor series for $h(x) = \dfrac{1}{1-x}$ about $a = -2$ and state the open interval of convergence of the series. (Hint: one way is to use the series for $\dfrac{1}{1-x}$.)

[5]

Question 5: Find the first three nonzero terms of the Maclaurin series for $f(x) = \cos(x) \cdot \ln(1 - x)$.

[5]

Question 6: Evaluate the following limit and state your answer as a single simplified fraction:

$$\lim_{x \to 0} \frac{e^{-x^2} + x^2 - 1}{\sin(3x^4)}$$

[5]