(1) Evaluate $\int_1^2 r^3 \ln(r)\,dr = I$

$u = \ln(r) \quad dv = r^3\,dr$

$du = \frac{1}{r}\,dr \quad v = \frac{r^4}{4}$

$$\therefore \; I = \int_1^2 u\,dv$$

$$= \left[uv\right]_1^2 - \int_1^2 v\,du$$

$$= \left[\frac{r^4 \ln(r)}{4}\right]_1^2 - \int_1^2 \frac{r^4}{4} \cdot \frac{1}{r}\,dr$$

$$= \frac{2^4 \ln(2)}{4} - \cancelto{0}{\frac{1^4 \ln(1)}{4}} - \frac{1}{16}\left[r^4\right]_1^2$$

$$= \boxed{4\ln(2) - \frac{15}{16}}$$

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(2) Determine $\int \frac{10}{(x-1)(x^2+9)}\,dx = I$

$$\frac{10}{(x-1)\underbrace{(x^2+9)}_{\text{irreducible}}} = \frac{A}{x-1} + \frac{Bx+C}{x^2+9}$$

$$= \frac{A(x^2+9)+(Bx+C)(x-1)}{(x-1)(x^2+9)}$$

$$= \frac{(A+B)x^2+(-B+C)x+(9A-C)}{(x-1)(x^2+9)}$$

$\therefore$ $A+B=0$ ① $\qquad$ ① $\Rightarrow$ $B=-A$

$-B+C=0$ ② $\qquad$ ② $\Rightarrow$ $C=B=-A$

$9A-C=10$ ③ $\qquad$ ③ $\Rightarrow$ $9A-(-A)=10 \Rightarrow 10A=10 \Rightarrow A=1$

$\therefore$ $A=1$, $B=-A=-1$, $C=-A=-1$

$$\therefore \quad I = \int \frac{1}{x-1} + \frac{-x-1}{x^2+9}\,dx$$

$$= \int \frac{1}{x-1} - \frac{1}{2}\underbrace{\int \frac{2x}{x^2+9}\,dx}_{\substack{u=x^2+9 \\ du=2x\,dx}} - \underbrace{\int \frac{1}{x^2+9}\,dx}_{\substack{\text{Formula \#18} \\ \text{with } a=3}}$$

$$= \boxed{\ln|x-1| - \frac{1}{2}\ln|x^2+9| - \frac{1}{3}\arctan\left(\frac{x}{3}\right) + C}$$

[8]