(1) Find the radius of convergence R and open interval of convergence $\mathcal{I}$ for the power series

$$\sum_{k=1}^{\infty} \frac{(-1)^k 3^{2k} (x-2)^k}{3k}$$

$$u_k(x) = \frac{(-1)^k 3^{2k} (x-2)^k}{3k}$$

$$\lim_{k\to\infty} \left| \frac{u_{k+1}(x)}{u_k(x)} \right| < 1$$

$$\Rightarrow \lim_{k\to\infty} \left| \frac{(-1)^{k+1} 3^{2k+2} (x-2)^{k+1}}{3k+3} \cdot \frac{3k}{(-1)^k 3^{2k} (x-2)^k} \right| < 1$$

$$\Rightarrow \lim_{k\to\infty} \underbrace{\left| \cancel{\frac{(-1)^{k+1}}{(-1)^k}} \right|}_{1} \cdot \underbrace{\left| \frac{3^{\cancel{2k}+2}}{3^{\cancel{2k}}} \right|}_{=9} \cdot \underbrace{\left| \frac{3k}{3k+3} \right|}_{\to 1} \cdot \underbrace{\left| \frac{(x-2)^{k+1}}{(x-2)^k} \right|}_{=|(x-2)|} < 1$$

$$\Rightarrow \quad 9|x-2| < 1$$

$$|x-2| < \frac{1}{9}$$

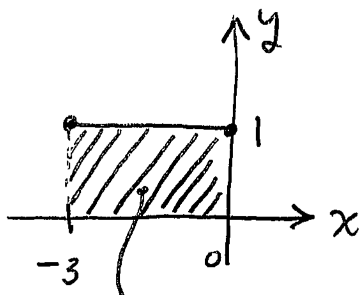
$$\therefore \; R = \frac{1}{9}, \quad \mathcal{I} = \left(2 - \frac{1}{9}, 2 + \frac{1}{9}\right)$$

$$= \boxed{\left(\frac{17}{9}, \frac{19}{9}\right)}$$

[7]

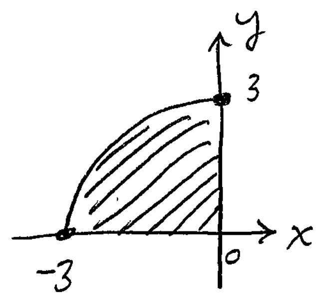
(2) Use an area interpretation to find $\int_{-3}^{0} \left(1+\sqrt{9-x^2}\right)\,dx = \int_{-3}^{0} 1\,dx + \int_{-3}^{0} \sqrt{9-x^2}\,dx$

$y = 1$:



$A_1 = (3)(1) = 3$

$y = \sqrt{9-x^2}$



$A_2 = \left(\frac{1}{4}\right)(\pi)(3^2)$

$= \frac{9\pi}{4}$

$\therefore \int_{-3}^{0} \left(1+\sqrt{9-x^2}\right)dx$

$= A_1 + A_2$

$= \boxed{3 + \frac{9\pi}{4}}$ or $\boxed{\frac{12+9\pi}{4}}$

[4]

(3) Evaluate $\int_{1}^{2} \left(\frac{x}{2} - \frac{2}{x}\right)\,dx$

$= \left[\frac{x^2}{4} - 2\ln|x|\right]_1^2$

$= \left(\frac{2^2}{4} - 2\ln|2|\right) - \left(\frac{1^2}{4} - \cancel{2\ln|1|}^{\,0}\right)$

$= \boxed{\frac{3}{4} - 2\ln(2)}$ or $\boxed{\frac{3 - 8\ln(2)}{4}}$

[4]