(1) For this question let $f(x) = \dfrac{4x^2+1}{x} = 4x + \dfrac{1}{x}$

(i) ~~Determine $T_1(x)$, the linear approximation to f at $a = 1$~~

$f(x) = 4x + \frac{1}{x}$; $f(1) = (4)(1) + \frac{1}{1} = 5$

$f'(x) = 4 - \frac{1}{x^2}$; $f'(1) = 4 - \frac{1}{1^2} = 3$

$\therefore T_1(x) = f(1) + f'(1)(x-1)$

$$\boxed{T_1(x) = 5 + 3(x-1)}$$

[3]

(ii) Use your approximation to estimate $f(1.2)$. State your answer as a single simplified fraction.

$f(1.2) \approx T_1(1.2)$

$= 5 + 3(1.2 - 1)$

$= 5 + 3\left(\frac{1}{5}\right)$

$= \dfrac{25+3}{5}$

$= \boxed{\dfrac{28}{5}}$

[3]

(iii) Determine an error bound on your approximation in part (ii). Again, state your answer as a single simplified fraction.

$f''(z) = \dfrac{2}{z^3}$, $a = 1$, $x = 1.2$

$|R_1(x)| = \left| \dfrac{f''(z)(x-a)^2}{2} \right|$

$= \left| \frac{1}{2} \cdot \frac{2}{z^3} \cdot (1.2-1)^2 \right|$

$\leq \left| \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{1^3} \cdot \left(\frac{1}{5}\right)^2 \right| = \boxed{\dfrac{1}{25}}$

[4]

(2) Let $f(x) = (x^2+1)e^x$. Use $T_2(x)$, a Maclaurin polynomial of degree two, to approximate $f(-1/4)$. State your answer as a single simplified fraction.
(note: there is no need to determine the error bound.)

$$f(x) = (x^2+1)e^x \quad ; \quad f(0) = 1$$

$$f'(x) = 2xe^x + (x^2+1)e^x ; \quad f'(0) = 1$$

$$f''(x) = 2e^x + 2xe^x + 2xe^x + (x^2+1)e^x ; \quad f''(0) = 3.$$

$$\therefore T_2(x) = f(0) + f'(0)x + \frac{f''(0)}{2}x^2$$

$$= 1 + x + \frac{3}{2}x^2$$

$$\therefore f(-\tfrac{1}{4}) \approx T_2(-\tfrac{1}{4})$$

$$= 1 - \frac{1}{4} + \frac{3}{2}\left(-\frac{1}{4}\right)^2$$

$$= \frac{32 - 8 + 3}{32}$$

$$= \boxed{\frac{27}{32}}$$

[5]