Question 1: Determine $\int \frac{2x^2+6x+7}{x(x^2+4x+7)}$

[10]

Question 2:

(a) Use T_4, the trapezoid rule on four subintervals, to approximate the integral $\int_0^2 \sin^2(\pi x)\, dx$.

[5]

(b) Determine an error bound in your approximation T_4 in part (a). Recall, the error in using the trapezoid rule to approximate $\int_a^b f(x)\, dx$ is at most $\dfrac{K(b-a)^3}{12n^2}$, where $|f''(x)| \leq K$ on $[a, b]$.

Hint: to determine K, you may find it easier if you think $\sin^2(\theta) = \dfrac{1-\cos(2\theta)}{2}$.

[5]

Question 3:

(a) Evaluate the following improper integral, making proper use of any required limits:

$$\int_0^{\ln 2} \frac{e^x}{\sqrt{e^x - 1}}\,dx$$

[5]

(b) Use the comparison theorem to decide if the following integral converges or diverges:

$$\int_1^{\infty} \frac{e^{-x}}{x^2 + \sqrt{x} + 1}\,dx$$

[5]

Question 4: Determine the area of the region in the first quadrant that is enclosed by the curves $y = 1 + x^2$, $y = x^2/4$, $y = 3 - x$, and the y-axis.

[5]

Question 5: The curve $y = \sqrt{r^2 - x^2}$ over $-r \leq x \leq r$ represents the top half of a circle of radius r centred at $(0, 0)$. Rotating the curve about the x-axis forms a sphere of radius r. Use the disk method to determine the volume of the sphere.

[5]

Question 6: The region in the first quadrant that is bounded by $y = x^2$, the vertical line $x = 1$ and the x-axis is rotated about the vertical line $x = 2$. Determine the volume of the resulting solid. Use either the washer method or cylindrical shells, whichever you prefer.

[5]

Question 7: The curve $y = \sqrt{1-(x-2)^2}$ over $1 \le x \le 3$ represents the top half of a circle of radius 1 centred at $(2, 0)$. If the region between the curve and x-axis is rotated about the y-axis, the resulting solid is the top half of a torus (a torus is essentially a mathematical doughnut, or bagel). Set up BUT DO NOT EVALUATE the integral representing the volume of the solid.

[5]