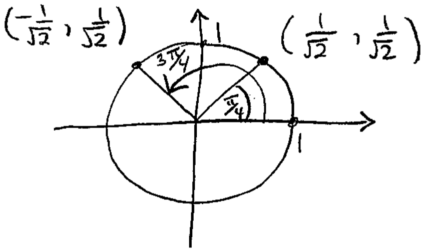
Question 1 [10 points]:

(a) Determine $\arccos(-1/\sqrt{2})$

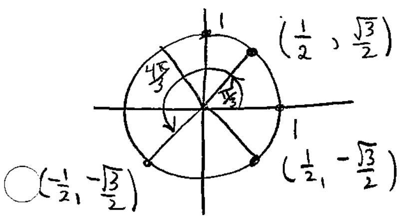
$\arccos\left(\frac{-1}{\sqrt{2}}\right)$ = angle θ in $[0, \pi]$ such that $\cos(\theta) = \frac{-1}{\sqrt{2}}$



$\therefore \arccos\left(\frac{-1}{\sqrt{2}}\right) = \boxed{\frac{3\pi}{4}}$

[2]

(b) Determine $\sin^{-1}(\sin(4\pi/3))$.



$\sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

$\sin^{-1}\left(\sin\left(\frac{4\pi}{3}\right)\right) = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

= angle $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ such that $\sin(\theta) = -\frac{\sqrt{3}}{2}$

$= \boxed{-\frac{\pi}{3}}$

[2]

(c) Determine $\lim_{x \to -\infty} \arctan\left(\frac{x+1}{x}\right)$.

As $x \to -\infty$, $\frac{x+1}{x} \to 1$,

$\therefore \lim_{x \to -\infty} \arctan\left(\frac{x+1}{x}\right) = \arctan(1)$

= angle $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ such that $\tan(\theta) = 1$

$= \boxed{\frac{\pi}{4}}$

[3]

(d) Calculate and simplify $f'(x)$ if $f(x) = x\cos^{-1}(x) - \sqrt{1-x^2}$.

$f'(x) = 1 \cdot \cos^{-1}(x) + \cancel{x\left(\frac{-1}{\sqrt{1-x^2}}\right) + \frac{\cancel{2}x}{\cancel{2}\sqrt{1-x^2}}}$

$= \boxed{\cos^{-1}(x)}$

[3]

Question 2 [10 points]:

(a) Simplifty $\cosh(\ln 3)$. Your final answer should be a simple fraction not containing any functions.

$$\cosh(\ln(3)) = \frac{e^{\ln(3)} + e^{-\ln(3)}}{2}$$

$$= \frac{e^{\ln(3)} + e^{\ln(3^{-1})}}{2}$$

$$= \frac{3 + \frac{1}{3}}{2}$$

$$= \frac{10}{6}$$

$$= \boxed{\frac{5}{3}}$$

[3]

(b) Determine $\lim_{x\to\infty} \frac{\cosh(x)\sinh(x)}{e^{2x}}$. $= \lim_{x\to\infty} \frac{e^x+e^{-x}}{2e^x} \cdot \frac{e^x - e^{-x}}{2e^x}$

$$= \lim_{x\to\infty} \frac{1}{4}(1 + \underbrace{e^{-2x}}_{\to 0})(1 - \underbrace{e^{-2x}}_{\to 0})$$

$$= \boxed{\frac{1}{4}}$$

[3]

(c) Determine $\lim_{x\to 0} \frac{\tanh(4x)}{\sinh(7x)}$. $\sim \frac{0}{0}$

$$\stackrel{H}{=} \lim_{x\to 0} \frac{4\,\operatorname{sech}^2(4x)}{7\cosh(7x)}$$

$$= \boxed{\frac{4}{7}}$$

[4]

Question 3 [10 points]: Find the following limits if they exist:

(a) $\lim_{x\to 0} \frac{x+\tan(x)}{\sin(2x)}$ $\sim$ "$\frac{0}{0}$"

$\overset{H}{=} \lim_{x\to 0} \frac{1+\sec^2(x)}{2\cos(2x)}$

$= \frac{1+1}{2} = \boxed{1}$

[3]

(b) $\lim_{x\to\infty} \frac{x^2}{e^x + x - 1}$ $\sim$ "$\frac{\infty}{\infty}$"

$\overset{H}{=} \lim_{x\to\infty} \frac{2x}{e^x+1}$ $\sim$ "$\frac{\infty}{\infty}$"

$\overset{H}{=} \lim_{x\to\infty} \frac{2}{e^x}$

$= \boxed{0}$

[3]

(c) $\lim_{x\to 0^+} \sqrt{x}\ln(x)$ $\sim$ "$0\cdot(-\infty)$"

$= \lim_{x\to 0^+} \frac{\ln(x)}{x^{-1/2}}$ $\sim$ "$\frac{-\infty}{\infty}$"

$\overset{H}{=} \lim_{x\to 0^+} \frac{\left(\frac{1}{x}\right)}{-\frac{1}{2}x^{-3/2}}$

$= \lim_{x\to 0^+} (-2)\left(\frac{1}{x}\right)\left(x^{3/2}\right)$

$= \lim_{x\to 0^+} -2x^{1/2}$

$= \boxed{0}$

[4]

Question 4 [5 points]: Find the following limit if it exists:

$$\lim_{x\to 0^+} x^{(x^2)} \sim \text{"}0^0\text{"}$$

$$x^{(x^2)} = e^{x^2 \ln(x)}$$

consider $\lim_{x\to 0^+} x^2 \ln(x) \sim 0\cdot(-\infty)$

$$= \lim_{x\to 0^+} \frac{\ln(x)}{x^{-2}} \sim \text{"}\frac{-\infty}{\infty}\text{"}$$

$$\stackrel{H}{=} \lim_{x\to 0^+} \frac{(1/x)}{-2x^{-3}}$$

$$= \lim_{x\to 0^+} -\frac{1}{2}x^2$$

$$= 0$$

$$\therefore \lim_{x\to 0^+} x^{(x^2)} = e^0 = \boxed{1}.$$

[5]

Question 5 [5 points]: Suppose an object moves with acceleration $a(t) = 2\sqrt{t} + \frac{1}{t^2}$ m/s^2 where $t > 0$ represents time in seconds. If velocity $v(1) = 1/3$ m/s and displacement $s(1) = 2$ m, determine a formula for $s(t)$, the displacement at time t.

$$a(t) = 2t^{1/2} + \frac{1}{t^2}$$

$$\therefore v(t) = \frac{4}{3}t^{3/2} - \frac{1}{t} + C_1$$

$$v(1) = \frac{1}{3} \Rightarrow \frac{4}{3}(1)^{3/2} - \frac{1}{1} + C_1 = \frac{1}{3}$$

$$\Rightarrow C_1 = \frac{1}{3} + 1 - \frac{4}{3} = 0$$

$$\therefore v(t) = \frac{4}{3}t^{3/2} - \frac{1}{t}.$$

$$\therefore s(t) = \frac{4}{3}\frac{t^{5/2}}{(5/2)} - \ln|t| + C_2$$

$$= \frac{8}{15}t^{5/2} - \ln|t| + C_2$$

$$s(1) = 2 \Rightarrow \frac{8}{15}(1)^{5/2} - \cancel{\ln|1|}^{\,0} + C_2 = 2$$

$$\Rightarrow C_2 = 2 - \frac{8}{15} = \frac{22}{15}$$

$$\boxed{\therefore s(t) = \frac{8}{15}t^{5/2} - \ln|t| + \frac{22}{15}}$$

[5]

Question 6 [10 points]: Find the most general antiderivative of each of the following functions:

(a) $f(x) = 2x + \dfrac{3}{1+x^2} + \cos(x)$

$$\boxed{F(x) = x^2 + 3\arctan(x) + \sin(x) + C}$$

[3]

(b) $f(x) = \dfrac{5 - 3x + x^3}{x} = 5\left(\frac{1}{x}\right) - 3 + x^2$

$$\boxed{F(x) = 5\ln|x| - 3x + \frac{x^3}{3} + C}$$

[3]

(c) $f(x) = \left(x + \frac{1}{x}\right)^2 = x^2 + 2 + x^{-2}$

$$\boxed{F(x) = \frac{x^3}{3} + 2x - \frac{1}{x} + C}$$

[4]