Math 122 - Quiz 3 **Name:** SOLUTIONS.

(1) Determime $\int \frac{36}{(x+5)^2(x-1)}\,dx = I$

$$\frac{36}{(x+5)^2(x-1)} = \frac{A}{x+5} + \frac{B}{(x+5)^2} + \frac{C}{x-1}$$

$$= \frac{A(x+5)(x-1) + B(x-1) + C(x+5)^2}{(x+5)^2(x-1)}$$

$$= \frac{(A+C)x^2 + (4A+B+10C)x + (-5A-B+25C)}{(x+5)^2(x-1)}$$

① $A + C = 0$

② $4A + B + 10C = 0$

③ $-5A - B + 25C = 36$

① $\Rightarrow$ $A = -C$

② $\Rightarrow$ $B = -4A - 10C = -4(-C) - 10C = -6C$

③ $\Rightarrow$ $-5(-C) - (-6C) + 25C = 36$

$36C = 36$

$\boxed{C = 1}$

$\therefore \boxed{A = -C = -1}$

$\therefore \boxed{B = -6C = -6}$

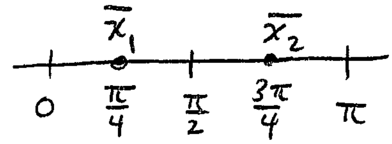
$$\therefore I = \int \frac{-1}{x+5} - \frac{6}{(x+5)^2} + \frac{1}{x-1}\,dx$$

$$= \boxed{-\ln|x+5| + \frac{6}{x+5} + \ln|x-1| + C}$$

[8]

(2) Use the midpoint rule with $n = 2$ subintervals to evaluate $\int_0^{\pi} x^2 \sin x \, dx$

$$\Delta x = \frac{\pi - 0}{2} = \frac{\pi}{2}$$



$$\bar{x}_1 = \frac{0 + \frac{\pi}{2}}{2} = \frac{\pi}{4}$$

$$\bar{x}_2 = \frac{\left(\frac{\pi}{2} + \pi\right)}{2} = \frac{3\pi}{4}$$

$$f(x) = x^2 \sin x$$

$$\therefore \int_0^{\pi} x^2 \sin x \, dx \approx M_2 = \left(\frac{\pi}{2}\right)\left[\left(\frac{\pi}{4}\right)^2 \sin\left(\frac{\pi}{4}\right) + \left(\frac{3\pi}{4}\right)^2 \sin\left(\frac{3\pi}{4}\right)\right]$$

$$= \left(\frac{\pi}{2}\right)\left[\frac{\pi^2}{4^2}\frac{1}{\sqrt{2}} + \frac{3^2\pi^2}{4^2}\frac{1}{\sqrt{2}}\right]$$

$$= \frac{10\pi^3}{32\sqrt{2}} = \boxed{\frac{5\pi^3}{16\sqrt{2}}}$$

[4]

---

(3) Evaluate the following improper integral making proper use of any required limits: $\int_1^{\infty} \frac{\ln x}{x} \, dx$

$$\int_1^{\infty} \frac{\ln x}{x} dx = \lim_{b \to \infty} \int_1^b \frac{\ln x}{x} dx \quad \left\} \begin{array}{l} u = \ln x \\ du = \frac{1}{x} dx \end{array} \right.$$

$$= \lim_{b \to \infty} \left[\frac{(\ln x)^2}{2}\right]_1^b$$

$$= \lim_{b \to \infty} \left[\frac{(\ln b)^2}{2} - \underbrace{\frac{(\ln 1)^2}{2}}_{0}\right]$$

$$= \boxed{\infty}$$

[3]