(1) Water flows from the bottom of a storage tank at a rate of $r(t) = 200 - 4t$ liters per minute, where $0 \leq t \leq 50$. Find the amount of water that flows from the tank during the first 10 minutes.

$$V = \int_0^{10} 200 - 4t \, dt$$

$$= \left[200t - \frac{4}{2} t^2 \right]_0^{10}$$

$$= \left[(200)(10) - 2(10)^2 \right] - \left[(200)(0) - 2(0)^2 \right]$$

$$= \boxed{1800 \text{ L}}$$

[4]

(2) Determine $\int \frac{(\ln x)^2}{x} \, dx = I$

Let $u = \ln(x)$

$du = \frac{1}{x} dx$

$$\therefore I = \int u^2 \, du$$

$$= \frac{u^3}{3} + C$$

$$= \boxed{\frac{(\ln(x))^3}{3} + C}$$

[3]

(3) Determine $\int_0^{\pi} \sec^2(t/4)\,dt = I$

Let $u = \frac{t}{4}$, $du = \frac{1}{4}dt$ $\Big\}$ $t=0 \Rightarrow u=0$, $t=\pi \Rightarrow u=\frac{\pi}{4}$

$$\therefore\ I = 4\int_0^{\frac{\pi}{4}} \sec^2(u)\,du$$

$$= 4\left[\tan(u)\right]_0^{\frac{\pi}{4}}$$

$$= 4\left(\cancelto{1}{\tan\left(\tfrac{\pi}{4}\right)} - \cancelto{0}{\tan(0)}\right)$$

$$= \boxed{4}$$

[4]

(4) Determine $\int x\cos(5x)\,dx = I$.

Let $u = x$ $\quad dv = \cos(5x)dx$

$du = dx \quad v = \frac{\sin(5x)}{5}$

$$I = \int u\,dv$$

$$= uv - \int v\,du$$

$$= \frac{x\sin(5x)}{5} - \int \frac{\sin(5x)}{5}\,dx$$

$$= \boxed{\frac{x\sin(5x)}{5} + \frac{\cos(5x)}{25} + C}$$

[4]