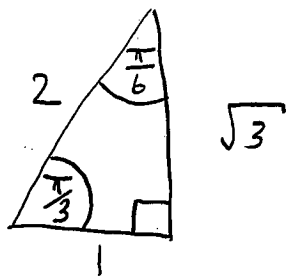


(1) [5] Determine the exact value of $\tan^{-1}(\sqrt{3})$.

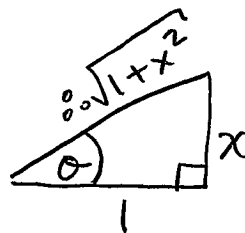


$$\therefore \tan^{-1}(\sqrt{3}) = \boxed{\frac{\pi}{3}}$$

(2) [5] Simplify $\sin(\tan^{-1}x)$. (Your final simplified answer should not contain any trigonometric or inverse trigonometric functions.)

$$\text{Let } \theta = \tan^{-1}(x)$$

$$\therefore \tan(\theta) = \frac{x}{1} \longrightarrow$$



$$\therefore \sin(\tan^{-1}(x)) = \sin(\theta) = \boxed{\frac{x}{\sqrt{1+x^2}}}$$

(3) [5] Determine $\lim_{x \rightarrow 0^-} \coth(x)$. (Recall that $\coth(x) = 1/\tanh(x)$.)

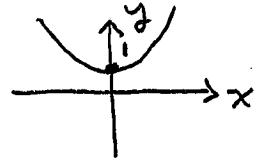
Solⁿ 1:

$$\lim_{x \rightarrow 0^-} \coth(x) = \lim_{x \rightarrow 0^-} \frac{\cosh(x)}{\sinh(x)}$$

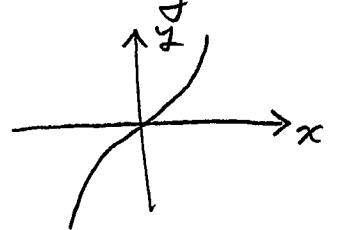
$$= \frac{1^+}{0^-}$$

$$= \boxed{-\infty}$$

since $y = \cosh(x)$:



since $y = \sinh(x)$:



Solⁿ 2:

$$\lim_{x \rightarrow 0^-} \coth(x) = \lim_{x \rightarrow 0^-} \frac{e^x + e^{-x}}{e^x - e^{-x}} \cdot \frac{e^x}{e^x}$$

$$= \lim_{x \rightarrow 0^-} \left. \frac{e^{2x} + 1}{e^{2x} - 1} \right\} \rightarrow 2$$

$$= \boxed{-\infty}$$