Here are some additional problems that review material for the final exam. I've tried to include questions from all of the major topics we have done, but this is by no means a complete list of all the material you should know for the final exam. The answers follow at the end.

1. Evaluate the following limits

   (a) $\lim_{x\to 0} \frac{\sinh x}{x}$ (b) $\lim_{x\to\infty} \tanh x$ (c) $\lim_{x\to\infty} \frac{\cosh x}{e^x}$

2. Differentiate the following functions.

   (a) $y = \ln(\sinh x)$
   (b) $y = \cosh(3x)\sinh(x)$
   (c) $y = \tanh(x)\sinh(2x)$
   (d) $y = \tanh(\cot x)$
   (e) $y = \arcsin(2x^2)$
   (f) $y = x^3 \arctan(e^x)$
   (g) $y = \operatorname{arcsec}(x^3)$
   (h) $y = e^x \arcsin(x^2)$
   (i) $y = \tan(\arccos x)$
   (j) $y = \coth^3(4x)$
   (k) $y = e^{\operatorname{csch} x}$
   (l) $y = \tanh^{-1}\sqrt{x}$
   (m) $y = \operatorname{sech}^{-1}(x^{-1})$
   (n) $y = (\sinh^{-1} x)^{3/2}$
   (o) $y = \ln(\tanh^{-1} x)$
   (p) $y = \arctan(\ln x)$
   (q) $y = \frac{\arctan x}{(1+x^2)^2}$

3. Find an expression for each of the following in terms of x.

   (a) $\cos(\arcsin x)$ (b) $\tan(\operatorname{arcsec} x)$ (c) $\cos(\arctan x)$

4. Verify the following formulas

   (a) $\cosh(x+y) = \cosh x \cosh y + \sinh x \sinh y$.
   (b) $\tanh x = \frac{e^{2x}-1}{e^{2x}+1}$
   (c) $\cosh x + \sinh x = e^x$

5. If $\cosh x = \frac{5}{3}$ and $x > 0$, find the values of the other hyperbolic functions at x.

6. Use implicit differentiation to find $\frac{dy}{dx}$ for the following equations.

   (a) $\arctan x + \arctan y = \frac{\pi}{2}$ (b) $(\arcsin x)(\arcsin y) = \frac{\pi^2}{16}$

7. (a) Find the Maclaurin polynomial of degree 3 for $f(x) = \tan x$.
   (b) Find $T_6(x)$ for $f(x) = \sqrt{x}$ centered at $x = 1$.
   (c) Find $T_5(x)$ for $f(x) = \arcsin x$ centered at $x = 0$.
   (d) Find $T_3(x)$ for $f(x) = (3x+2)^{1/3}$ centered at $x = 2$.

8. Find the Taylor series for the given function centered at a.

   (a) $f(x) = \frac{1}{1-2x}$, $a = 0$
   (b) $f(x) = e^{x-1}$, $a = 0$
   (c) $f(x) = \frac{1}{x}$, $a = 1$
   (d) $f(x) = \frac{1}{1-x}$, $a = 5$
   (e) $f(x) = e^{3x}$, $a = -1$
   (f) $f(x) = \frac{1}{1-x^2}$, $a = 3$

9. Use a Taylor series to determine the following limits.

   (a) $\lim_{x \to 0} \dfrac{\cos x^2 - 1}{x^4}$

   (b) $\lim_{x \to 1} \dfrac{\ln x - (x-1)}{(x-1)^2}$

   (c) $\lim_{x \to 0} \dfrac{e^x - 1}{x}$

10. (a) Use $T_3(x)$ centered at $x = 0$ to estimate $\displaystyle\int_{-1}^{1} \frac{\sin x}{x}\, dx$

    (b) Use $T_5(x)$ centered at $x = 0$ to estimate $\displaystyle\int_{-1}^{1} e^{-x^2}\, dx$

    (c) Use $T_5(x)$ centered at $x = 1$ to estimate $\displaystyle\int_{1}^{2} \ln x\, dx$

11. Use a known Taylor series to find the Taylor series for the following functions centered at $x = 0$.

    (a) $f(x) = e^{-3x}$

    (b) $f(x) = xe^{-x^2}$

    (c) $f(x) = x\sin(2x)$

    (d) $f(x) = \frac{1-\cos(x^2)}{x}$

12. Use the Binomial Theorem to find the Maclaurin series for the following functions

    (a) $f(x) = \frac{1}{\sqrt{1-x}}$

    (b) $f(x) = \frac{6}{\sqrt[3]{1+3x}}$

13. Determine if the following differential equation is separable.

    (a) $y' = (3x+1)\cos y$

    (b) $y' = (3x+y)\cos y$

    (c) $y' = x^2y + y\cos x$

    (d) $y' = x^2y - x\cos y$

14. Which of the following functions are solutions to the differential equation $y'' + y = \sin x$?

    (a) $y = \sin x$ (b) $y = \cos x$ (c) $y = \frac{1}{2}x\sin x$ (d) $y = -\frac{1}{2}x\cos x$

15. For what values of k does $y = 5 + 3e^{kx}$ satisfy the differential equation $\frac{dy}{dx} = 10 - 2y$?

16. Find the general solution to the following differential equations, in an explicit form if possible.

    (a) $y' = (x^2+1)y$

    (b) $y' = 2x^2y^2$

    (c) $y' = \frac{2xe^y}{ye^x}$

    (d) $y' = y^2 - y$

    (e) $y' = \frac{\cos^2 y}{4x-3}$

    (f) $y' = \frac{xy}{1+x^2}$

17. Solve the following initial value problems, explicitly if possible.

(a) $y' = 3(x+1)^2 y$, $y(0) = 1$

(b) $y' = \frac{4y}{x+3}$, $y(-2) = 1$

(c) $y' = \dfrac{4x}{\cos y}$, $y(0) = 0$

(d) $yy' = xe^{-y^2}$, $y(0) = -1$

18. Find an equation of the curve that passes through the point $(0, 1)$ and whose slope at (x, y) is xy.

19. Evaluate the following limits.

(a) $\displaystyle\lim_{x\to -2} \frac{x^2+6x+8}{x^2-3x-10}$

(b) $\displaystyle\lim_{x\to \frac{\pi}{2}} \frac{\ln(\sin x)^3}{\frac{\pi}{2}-x}$

(c) $\displaystyle\lim_{x\to 0} \frac{e^x - \ln(1+x) - 1}{x^2}$

(d) $\displaystyle\lim_{x\to 0^+} \frac{1-\cos x - x\sin x}{2 - 2\cos x - \sin^2 x}$

(e) $\displaystyle\lim_{x\to \frac{\pi}{2}} \frac{3\sec x + 5}{\tan x}$

(f) $\displaystyle\lim_{x\to 0} x\ln(x^{1000})$

(g) $\displaystyle\lim_{x\to 0}(\csc^2 x - \cot^2 x)$

(h) $\displaystyle\lim_{x\to 0^+} (3x)^{x^2}$

(i) $\displaystyle\lim_{x\to 0}(x + e^{x/3})^{3/x}$

(j) $\displaystyle\lim_{x\to \frac{\pi}{2}} (\sin x)^{\cos x}$

(k) $\displaystyle\lim_{x\to 0^+} (\tan x)^{2/x}$

(l) $\displaystyle\lim_{x\to 0} \left(\csc x - \frac{1}{x}\right)$

(m) $\displaystyle\lim_{x\to 0^+} (1+2e^x)^{1/x}$

20. Determine if the following integral converges or diverges. Find the value of the integral if it converges.

(a) $\displaystyle\int_1^\infty 2xe^{-x^2}\, dx$

(b) $\displaystyle\int_9^\infty \frac{x}{\sqrt{1+x^2}}\, dx$

(c) $\displaystyle\int_e^\infty \frac{1}{x\ln x}\, dx$

(d) $\displaystyle\int_{-\infty}^1 \frac{1}{(2x-3)^3}\, dx$

(e) $\displaystyle\int_{-\infty}^\infty \frac{x}{\sqrt{x^2+9}}\, dx$

(f) $\displaystyle\int_0^\infty e^{-x}\cos x\, dx$

(g) $\displaystyle\int_3^{10} \frac{1}{\sqrt{x-3}}\, dx$

(h) $\displaystyle\int_{-1}^{128} x^{-5/7}\, dx$

(i) $\displaystyle\int_0^4 \frac{1}{(2-3x)^{1/3}}\, dx$

(j) $\displaystyle\int_{-2}^{-1} \frac{1}{(x+1)^{4/3}}\, dx$

(k) $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\sin x}{1-\cos x}\, dx$

(l) $\displaystyle\int_0^{\pi} \frac{1}{\cos x - 1}\, dx$

(m) $\displaystyle\int_1^e \frac{1}{x\ln x}\, dx$

21. Find the area of the region between the curves $y = (x-8)^{-2/3}$ and $y = 0$ over the interval $[0, 8)$.

22. Find the area of the region under the curve $y = \frac{2}{4x^2-1}$ to the right of $x = 1$.

23. Use the Comparison Test to determine if the integral converges or diverges.

(a) $\displaystyle\int_0^\infty \frac{x}{1+x^3}\, dx$

(b) $\displaystyle\int_1^\infty \frac{x+1}{\sqrt{x^4-x}}\, dx$

(c) $\displaystyle\int_0^1 \frac{\sec^2 x}{x\sqrt{x}}\, dx$

(d) $\displaystyle\int_3^\infty \frac{\ln x}{x}\, dx$

(e) $\displaystyle\int_1^\infty \frac{1}{\sqrt{x^6+x}}\, dx$

24. For what values of c does $\displaystyle\int_0^\infty xe^{cx}\, dx$ converge?

25. For what values of k does $\int_0^1 x^k \ln x \, dx$ converge?

26. Find the area between the given curves.

   (a) $y = \frac{1}{4}(x^2 - 7)$, $y = 0$ over the interval $[0, 2]$
   (b) $y = (x - 3)(x - 1)$, $y = x$
   (c) $x = 8y - y^2$, $x = 0$
   (d) $x = -6y^2 + 4y$, $x + 3y = 2$
   (e) $4y^2 - 2x = 0$, $4y^2 + 4x - 12 = 0$
   (f) $y = x + 6$, $y = x^3$, $2y + x = 0$
   (g) $x = y$, $x = -y$, $x = 1$
   (h) $y = x$, $y = 2$, $y = 6 - x$, $y = 0$

27. Find k such that the area bounded by the curves $y = x^2$ and $y = k - x^2$ is 72.

28. The base of a solid is the region inside the circle $x^2 + y^2 = 4$. The cross sections perpendicular to the x-axis are squares. Find the volume of the solid.

29. The base of a solid is the region bounded by $y = \sqrt{x}$ and $y = x^2$. The cross sections perpendicular to the x-axis are semi-circles. Find the volume of the solid.

30. The base of a solid is the region bounded by $y = 1 - x^2$ and the x-axis. The cross sections perpendicular to the x-axis are isosceles triangles with height equal to the base. Find the volume of the solid.

31. The base of a solid is the region enclosed by $y = x^2$ and $y = 3$. The cross sections perpendicular to the y-axis are squares. Find the volume of the solid.

32. Use the disc/washer method to find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line.

   (a) $y = x^2$, $y = 4$, $x = 0$; about the y-axis
   (b) $y = \frac{1}{\sqrt{x}}$, $y = 0$, $x = 1$, $x = 5$; about the x-axis
   (c) $y = 6 - x^2$, $y = 2$; about the y-axis
   (d) $y = x^2$, $x = y^2$; about the line $y = -2$
   (e) $y = x^2$, $x = y^2$; about the line $x = 3$

33. Find the length of the given curve.

   (a) $y = \frac{2}{3}(x^2 + 1)^{3/2}$, $0 \le x \le 2$
   (b) $y = \frac{1}{6}x^3 + \frac{1}{2x}$, $1 \le x \le 3$
   (c) $8x^2y - 2x^6 = 1$ from $(1, 3/8)$ to $(2, 192/32)$
   (d) $y = \ln(\sin x)$, $\frac{\pi}{4} \le x \le \frac{\pi}{2}$

34. Calculate the length of the astroid $x^{2/3} + y^{2/3} = 1$.

35. A tank has the shape of a frustum of a cone. The top radius is 6 ft, the bottom radius is 3 ft and the tank is 8 ft high. The tank is full of water. Find the work required to pump the water out of the top of the tank. (The density of water is 62.5 lb/ft^3.)

36. A banner in the shape of an isosceles triangle is hung from the roof over the side of a building. The triangle has a base of 25 ft and a height of 20 ft. The banner is made from material with a density of 5 lbs/ft^2. Find the work required to pull the banner onto the roof of the building.

37. A cable weighing 2 lb/ft is used to haul a 200 lb load to the top of a shaft that is 500 feet deep. How much work is done?

38. Set up a Riemann sum to find the exact area under the curve, $y = 2x^2 + 1$ on $[1, 3]$ using the right endpoint.

39. Compute the Midpoint, Trapezoid and Simpson's rule with $n = 4$ for the following. On an appropriate graph, draw a representation of the Midpoint and Trapezoid rule.

    (a) $\displaystyle\int_1^3 \frac{1}{x^2}\,dx$

    (b) $\displaystyle\int_0^2 \sqrt{x}\,dx$

40. How large should n be so that the error in using the Trapezoid rule in estimating $\displaystyle\int_1^4 \sqrt{x}\,dx$ is less than 0.01?

41. Consider the integral $\displaystyle\int_0^{\pi/2} \cos x\,dx$.

    (a) Calculate M_6 to estimate this integral. Is M_6 an overestimate or underestimate.

    (b) Find a bound on the error in using M_6 to estimate this integral.

42. Consider the integral $\displaystyle\int_0^1 e^{-2x}\,dx$.

    (a) Calculate S_6 to estimate this integral.

    (b) Find a bound on the error in using S_6 to estimate this integral.

43. Consider the integral $\displaystyle\int_1^5 \ln x\,dx$.

    (a) Find a bound on the error in using S_8 to estimate this integral.

    (b) How large should n be so that the error in using S_8 to estimate this integral is at most 10^{-6}.

44. Find the derivative $f'(x)$ for each of the following.

    (a) $\displaystyle f(x) = \int_0^{x^2} \cos(3t) + 1\,dt$

    (b) $\displaystyle f(x) = \int_x^{-1} \sqrt{t^2+1}\,dt$

    (c) $\displaystyle f(x) = \int_2^{1/x} \arctan t\,dt$

    (d) $\displaystyle f(x) = \int_0^{\tan x} \sqrt{t+\sqrt{t}}\,dt$

(e) $f(x) = \int_{\sqrt{x}}^{x^3} \sqrt{t} \sin t \, dt$

45. Use the Fundamental Theorem of Calculus to find an antiderivative of $\sin \sqrt{x^2+1}$.

46. Find the average value of $f(x) = 2x^3 - 3x^2$ on the interval $[-1, 3]$.

47. The linear density of a rod 8 m long is $\frac{12}{\sqrt{x+1}}$ kg/m, where x is measured in meters from one end of the rod. Find the average density of the rod.

48. Find the number b such that the average value of $f(x) = 2 + 6x - 3x^2$ on the interval $[0, b]$ is 3.

49. Use the table of integrals to evaluate the following:

(a) $\int \frac{x^4}{\sqrt{x^{10} - 2}} \, dx$

(b) $\int \frac{\sin x \cos x}{\sqrt{1 + \cos x}} \, dx$

(c) $\int \frac{\sin^2 x \cos x}{\sqrt{\sin^2 x + 4}} \, dx$

(d) $\int \frac{x^2}{\sqrt{16x^2 + 9}} \, dx$

(e) $\int e^x \sqrt{9 + e^{2x}} \, dx$

50. Use the method of partial fractions to evaluate the following integrals.

(a) $\int \frac{3x - 13}{x^2 + 3x - 10} \, dx$

(b) $\int \frac{2x^2 + x - 4}{x^3 - x^2 - 2x} \, dx$

(c) $\int \frac{x^3}{x^2 + x - 2} \, dx$

(d) $\int \frac{2x^2 + x - 8}{x^3 + 4x} \, dx$

(e) $\int \frac{5x^2 + 3x - 2}{x^3 + 2x^2} \, dx$

51. Use a trig substitution to evaluate the following integrals.

(a) $\int \frac{x^2}{\sqrt{9 - x^2}} \, dx$

(b) $\int \frac{x^3}{\sqrt{9 + x^2}} \, dx$

(c) $\int \frac{1}{\sqrt{x^2 - 4}} \, dx$, $x > 2$

(d) $\int \frac{x}{(3 - 2x - x^2)^{3/2}} \, dx$

(e) $\int_{\sqrt{2}}^{2} \frac{1}{x^3 \sqrt{x^2 - 1}} \, dx$

(f) $\int \frac{\sqrt{1 + x^2}}{x} \, dx$

(g) $\int_0^{0.6} \frac{x^2}{\sqrt{9 - 25x^2}} \, dx$

52. Evaluate the following integrals.

(a) $\int_1^2 \frac{\sqrt{\ln x}}{x} \, dx$

(b) $\int \frac{x^2}{x^3 + 1} \, dx$

(c) $\int \tan(2x) \, dx$

(d) $\int_0^1 \frac{e^x - 1}{e^{2x}} \, dx$

(e) $\int \frac{2}{\sqrt{x} e^{\sqrt{x}}} \, dx$

(f) $\displaystyle\int_1^{\sqrt{3}} \frac{1}{2+2x^2}\,dx$

(g) $\displaystyle\int \frac{2x^2}{\sqrt{1-x^6}}\,dx$

(h) $\displaystyle\int \frac{\cos x}{4+\sin^2 x}\,dx$

(i) $\displaystyle\int_0^{1/4} \frac{3}{\sqrt{1-4x^2}}\,dx$

(j) $\displaystyle\int_0^1 \frac{\cosh(2x)}{3+\sinh(2x)}\,dx$

(k) $\displaystyle\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx$

(l) $\displaystyle\int \frac{4}{5+2x+x^2}\,dx$

(m) $\displaystyle\int \frac{x+1}{\sqrt{3-2x-x^2}}\,dx$

(n) $\displaystyle\int_{-1}^0 e^x\cot(e^x)\csc(e^x)\,dx$

(o) $\displaystyle\int \coth x\ln(\sinh x)\,dx$

53. Evaluate the following integrals.

(a) $\displaystyle\int \sin^5(4x)\cos^2(4x)\,dx$

(b) $\displaystyle\int \sin(4y)\cos(5y)\,dy$

(c) $\displaystyle\int \sec^6 x\tan^2 x\,dx$

(d) $\displaystyle\int \frac{\sin^3 x}{\cos^4 x}\,dx$

(e) $\displaystyle\int \sqrt{\tan x}\sec^4 x\,dx$

(f) $\displaystyle\int_{\pi^2/16}^{\pi^2/9} \frac{2^{\sin\sqrt{x}}\cos\sqrt{x}}{\sqrt{x}}\,dx$

(g) $\displaystyle\int \sec^3(5x)\tan^3(5x)\,dx$

(h) $\displaystyle\int x\arcsin x\,d$

(i) $\displaystyle\int x^5e^{-x^2}\,dx$

(j) $\displaystyle\int \sin(\ln x)\,dx$

(k) $\displaystyle\int x^3\ln x\,dx$

(l) $\displaystyle\int x\csc^2 x\,dx$

(m) $\displaystyle\int x^5\sqrt{x^3+4}$

(n) $\displaystyle\int x\cosh x\,dx$

(o) $\displaystyle\int x2^x\,dx$

(p) $\displaystyle\int e^{2x}\sin(3x)\,dx$

(q) $\displaystyle\int e^{6x}\sin(e^{2x})\,dx$

(r) $\displaystyle\int \arccos x\,dx$

(s) $\displaystyle\int_1^e \sqrt{t}\ln t\,dt$

54. Use a triangle to simplify the expression $\tan(\arccos(3/5))$.

55. Find all solutions to $\sec(2x)=2$.

56. Find all the errors in the string. Then, determine the correct value of the limit.

$$\lim_{x\to 0}\frac{x^2}{\ln x^2}=\lim_{x\to 0}\frac{x^2}{2\ln x}=\lim_{x\to 0}\frac{2x}{2/x}=\lim_{x\to 0}\frac{2}{-2/x^2}=\lim_{x\to 0}-x^2=0$$

57. Use the cylindrical shell method to find the the volume of the solid obtained by rotating the region bounded by the given curves about the specified line.

(a) $y=x^2$, $y=8-x^2$; about the y-axis

(b) $y=x$, $x+2y=3$, $y=0$; about the x-axis

(c) $y=4x-x^3$, $y=0$; about the y-axis

(d) $y=x-x^3$, $y=0$ in the first quadrant; about the line $x=2$

(e) $y=x^2$, $y=0$, $x=1$, $x=-1$; about the line $x=2$

(f) $y=x^2+2$, $y=0$, $x=0$, $x=2$; about the line $y=-2$

58. Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line. (You must decide which method to use.)

(a) $y = x^2$, $x = y^2$; about the x-axis

(b) $y = x - x^3$, $y = 0$ in the first quadrant; about the line $y = -1$

(c) $y = x^3$, $y = 0$, $x = 2$; about the line $x = 3$

(d) $y = e^{-x^2}$, $y = 0$, $x = 0$, $x = 1$; about the y-axis

59. If 5 J of work are needed to stretch a spring 10 cm beyond its natural length, how much work is required to stretch the spring 15 cm beyond its natural length?

60. A hemispherical tank with radius 10 m is sitting on the ground on its curved end. The tank is full of water. Find the work required to pump all of the water to a point 2 meters above the top of the tank. (The density of water is 1000 kg/m^3.)

61. An inverted conical tank with radius 5 meters and height 10 meters is half full of water. Find the work required to pump all of the water to a point 2 meters above the top of the tank. (The density of water is 1000 kg/m^3.)

62. A 20 foot chain with density 3 lb/ft is coiled on the ground. How much work is done in lifting this chain so that it is fully extended and one end touches the ground.

63. A plate in the shape of an equilateral triangle with base 2 meters is submerged vertically in a tank of oil so that the tip of the triangle is at the surface of the oil. The density of the oil is 900 kg/m^3. Find the force of the oil on the plate.

64. A water trough 10 feet long has a trapezoid cross section. The trapezoid is 3 feet high, 2 feet wide at the bottom and 4 feet wide at the top. If the trough is full of water, find the force of the water on one end of the trough. (The density of water is 62.4 lb/ft^3.)

65. A gate in the vertical face of a dam is the shape of an isosceles triangle that is 5 feet high and 8 feet wide. Find the force of the water on the gate if the top of the gate is 10 feet beneath the surface of the water. (The tip of the triangle is pointing down.)

66. Determine the function $g(x)$ and all values of c such that $\displaystyle\int_c^x g(t)\,dt = x^2 + x - 6$.

67. Find $G(1)$, $G'(0)$ and $G'(\pi/4)$ where $G(x) = \displaystyle\int_1^x \tan t\,dt$.

68. Evaluate the following integrals by interpreting each in terms of area.

(a) $\displaystyle\int_0^5 \sqrt{25 - x^2}\,dx$

(b) $\displaystyle\int_{-2}^2 2 - |x|\,dx$

Answers

1. (a) 1 (b) 1 (c) $\frac{1}{2}$

2. (a) $\coth x$

(b) $\cosh(3x)\cosh(x) + 3\sinh(3x) + \sinh(x)$

(c) $2\tanh(x)\cosh(2x) + \sinh(2x)\,\text{sech}^2(x)$

(d) $-\csc^2 x\,\text{sech}^2(\cot x)$

(e) $\frac{4x}{\sqrt{1-4x^4}}$

(f) $x^2\left(\frac{xe^x}{1+e^{2x}} + 3\arctan(e^x)\right)$

(g) $\frac{3}{|x|\sqrt{x^6-1}}$

(h) $e^x \arcsin(x^2) + \frac{2xe^x}{\sqrt{1-x^4}}$

(i) $-\frac{\sec^2(\arccos x)}{\sqrt{1-x^2}}$

(j) $-12\coth^2(4x)\,\text{csch}^2(4x)$

(k) $-e^{\text{csch}\,x}(\text{csch}\,x\coth x)$

(l) $\frac{1}{2(1-x)\sqrt{x}}$

(m) $\frac{x}{|x|\sqrt{x^2-1}}$

(n) $\frac{3\sqrt{\sinh^{-1}x}}{2\sqrt{1+x^2}}$

(o) $\frac{1}{(1-x^2)\tanh^{-1}x}$

(p) $\frac{1}{x+x(\ln x)^2}$

(q) $\frac{1-4x\arctan x}{(1+x^2)^3}$

3. (a) $\sqrt{1-x^2}$ (b) $\sqrt{x^2-1}$ for $x \geq 1$ (c) $\frac{1}{1+x^2}$

4.

5. $\text{sech}\,x = \frac{3}{5}$, $\sinh x = \frac{4}{3}$, $\text{csch}\,x = \frac{3}{4}$, $\tanh x = \frac{4}{5}$, $\coth x = \frac{5}{4}$

6. (a) $\frac{dy}{dx} = \frac{-1-y^2}{1+x^2}$

(b) $\frac{dy}{dx} = \frac{-\arcsin x\sqrt{1-y^2}}{\arcsin x\sqrt{1-x^2}}$

7. (a) $x + \frac{x^3}{3}$

(b) $1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \frac{1}{16}(x-1)^3 - \frac{5}{128}(x-1)^4 + \frac{7}{256}(x-1)^5 - \frac{21}{1024}(x-1)^6$

(c) $x + \frac{1}{6}x^3 + \frac{3}{40}x^5$

(d) $2 + \frac{1}{4}(x-2) - \frac{1}{32}(x-2)^2 + \frac{5}{768}(x-2)^3$

8. (a) $1 + 2x + 2^2x^2 + 2^3x^3 + \cdots$

(b) $\frac{1}{e^2} + \frac{1}{e^2}x + \frac{1}{2e^2}x^2 + \frac{1}{3!e^2}x^3 + \cdots$

(c) $1 - (x-1) + (x-1)^2 - (x-1)^3 + \cdots$

(d) $-\frac{1}{4} + \frac{4^2}{(}x-5) - \frac{1}{4^3}(x-5)^2 + \frac{1}{4^4}(x-5)^3 + \cdots$

(e) $e^{-3} + 3e^{-3}(x+1) + \frac{3e^{-3}}{2}(x+1)^2 + \frac{3^2e^{-3}}{3!}(x+1)^3 \cdots$

(f) $-\frac{1}{2^3} + \frac{2^2-1}{2^5}(x-3) - \frac{2^3-1}{2^7}(x-3)^2 + \frac{2^4-1}{2^9}(x-3)^3 - \cdots$

9. (a) $-\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 1

10. (a) $\frac{1703}{900}$ (b) $\frac{5651}{3780}$ (c) $\frac{2}{5}$

11. (a) $1 - 3x + \frac{3^2}{2}x^2 - \frac{3^3}{3!} + \frac{3^4}{4!}x^4 - \cdots$

(b) $x - x^2 + \frac{1}{2}x^3 - \frac{1}{3!}x^4 + \cdots$

(c) $2x - \frac{2^3}{3!}x^3 + \frac{2^5}{5!}x^5 - \frac{2^7}{7!}x^7 + \cdots$

(d) $-\frac{1}{2}x^3 + \frac{1}{4!}x^7 - \frac{1}{6!}x^{11} + \frac{1}{8!}x^{15} - \cdots$

12. (a) $1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{5}{16}x^3 + \frac{35}{128}x^4 + \cdots$

(b) $6 - 6x + 12x^2 - 28x^3 + 70x^4 + \cdots$

13.

(a) yes
(b) no
(c) yes
(d) no

14. (d)

15. -2

16. (a) $y = Ae^{x+x^3/3}$
(b) $y = -\left(\frac{2}{3}x^3 + C\right)^{-1}$
(c) $(y+1)e^{-y} = 2(x+1)e^{-x} + C$
(d) $y = \dfrac{1}{1+ce^x}$
(e) $y = \arctan\left(\frac{1}{4}\ln|4x-3| + c\right)$
(f) $y = c\sqrt{1+x^2}$

17. (a) $y = e^{-1}e^{(x+1)^3}$
(b) $y = (x+3)^4$
(c) $\sin y = 2x^2$
(d) $y = -\sqrt{\ln(x^2+e)}$

18. $y = e^{x^2/2}$

19. (a) $-\frac{2}{7}$
(b) 0
(c) 1
(d) $-\infty$
(e) 3
(f) 0
(g) 1
(h) 1
(i) e^4
(j) 1
(k) 0
(l) 0
(m) ∞

20. (a) $\frac{1}{e}$
(b) diverges
(c) diverges
(d) $-\frac{1}{4}$
(e) diverges
(f) $\frac{1}{2}$
(g) $2\sqrt{7}$
(h) $\frac{21}{2}$
(i) $\frac{1}{2}(2^{2/3} - 10^{2/3})$
(j) diverges
(k) diverges
(l) diverges
(m) diverges

21. 6

22. $\frac{\ln 3}{2}$

23. (a) converges
(b) diverges
(c) diverges
(d) diverges
(e) converges

24. $c < 0$

25. $k > -1$

26. (a) $\frac{17}{6}$
(b) $\frac{13\sqrt{13}}{6}$
(c) $\frac{256}{3}$
(d) $\frac{1}{216}$
(e) 4
(f) 22
(g) 1
(h) 8

27. 18

28. $\frac{128}{3}$

29. $\frac{9\pi}{280}$

30. $\frac{8}{15}$

31. 18

32. (a) 8π (b) $\pi \ln 5$ (c) 8π (d) $\frac{49\pi}{30}$ (e) $\frac{17\pi}{10}$

33. (a) $\frac{22}{3}$ (b) $\frac{14}{3}$ (c) $\frac{123}{32}$ (d) $\ln(1+\sqrt{2})$

34. 6

35. 1.04×10^5 ft-lb

36. $\frac{25000}{3}$ ft-lb

37. $350,000$ ft-lb

38. $\lim_{n\to\infty} \sum_{i=1}^{n} \left(2\left(\frac{n+2i}{n}\right)^2 + 1\right)\frac{2}{n}$

39. (a) 0.6481464407, 0.705, 0.6670976271 (b) 1.903467524, 1.819479217, 1.875471422

40. 8

41. (a) 1.0028615; overestimate (b) 0.00448586

42. (a) 0.432361 (b) 6.85871×10^{-5}

43. (a) 0.0083333 (b) 78

44. (a) $2x\cos(3x^2) + 2x$ (b) $-\sqrt{x^2+1}$ (c) $-\frac{\arctan(1/x)}{x^2}$ (d) $\sqrt{\tan x + \sqrt{\tan x}}\sec^2 x$ (e) $3x^{7/2}\sin(x^3) - \frac{\sin\sqrt{x}}{2x^{1/4}}$

45. $\int_0^x \sin\sqrt{t^2+1}\,dt$

46. 3

47. 6 kg/m

48. $\frac{3+\sqrt{5}}{2}$

49. (a) $\frac{1}{5}\ln|x^5 + \sqrt{x^{10}-2}| + C$
(b) $-\frac{2}{3}(\cos x - 2)\sqrt{1+\cos x} + C$
(c) $\frac{1}{2}\sin x\sqrt{4+\sin^2 x} - 2\ln(\sin x + \sqrt{4+\sin^2 x}) + C$

(d) $\frac{1}{32}x\sqrt{16x^2+9} - \frac{9}{128}\ln(4x+\sqrt{16x^2+9}) + C$
(e) $\frac{1}{2}e^x\sqrt{9+e^{2x}} + \frac{9}{2}\ln(e^x+\sqrt{9+e^{2x}}) + C$

50. (a) $4\ln x + 5 - \ln|x-2| + C$
(b) $2\ln|x| - \ln|x+1| + \ln|x-2| + C$
(c) $\frac{1}{2}x^2 - x + \frac{8}{3}\ln|x+2| + \frac{1}{3}\ln|x-1| + C$
(d) $-2\ln|x| + \frac{1}{2}\arctan(x/2) + 2\ln|x^2+4| + C$
(e) $2\ln|x| + \frac{1}{x} + 3\ln|x+2| + C$

51. (a) $\frac{9}{2}\arcsin(x/3) - \frac{1}{2}\sqrt{9-x^2} + C$
(b) $\frac{1}{3}(9+x^2)^{3/2} - 9\sqrt{9+x^2} + C$
(c) $\frac{1}{2}\operatorname{arcsec}(x/2) + C$
(d) $\frac{3-x}{4\sqrt{3-2x-x^2}} + C$
(e) $\frac{\pi}{24} + \frac{\sqrt{3}}{8} + C$
(f) $\ln|(\sqrt{1+x^2}-1)/x| + \sqrt{1+x^2} + C$
(g) $\frac{9\pi}{500}$

52. (a) $\frac{2}{3}(\ln 2)^{3/2}$
(b) $\frac{1}{3}\ln|x^3+1| + C$
(c) $-\frac{1}{2}\ln|\tan(2x)| + C$
(d) $-e^{-1} + \frac{1}{2}e^{-2} + \frac{1}{2}$
(e) $-4e^{-\sqrt{x}} + C$
(f) $\frac{\pi}{4}$
(g) $\frac{2}{3}\arcsin(x^3) + C$
(h) $\frac{1}{2}\arctan\left(\frac{\sin x}{2}\right) + C$
(i) $\frac{\pi}{4}$
(j) $\frac{1}{2}\ln|3+\sinh(2)| - \frac{1}{2}\ln|3|$
(k) $\arcsin(e^x) + C$
(l) $2\arctan\left(\frac{x+1}{2}\right) + C$
(m) $-\sqrt{3-2x-x^2} + C$
(n) $-\csc(1) + \csc(e^{-1})$
(o) $\frac{1}{2}(\ln(\sinh x))^2 + C$

53. (a) $-\frac{1}{12}\cos^3(4x) + \frac{1}{10}\cos^5(4x) - \frac{1}{28}\cos^7(4x) + C$
(b) $-\frac{1}{18}\cos(9y) + \frac{1}{2}\cos(-y) + C$
(c) $\frac{1}{3}\tan^3 x + \frac{2}{5}\tan^5 x + \frac{1}{7}\tan^7 x + C$
(d) $\frac{1}{3\cos^3 x} - \frac{1}{\cos x} + C$
(e) $\frac{2}{3}(\tan x)^{3/2} + \frac{2}{7}(\tan x)^{7/2} + C$
(f) $\frac{2}{\ln 2}(2^{\sqrt{3}/2} - 2^{\sqrt{2}/2})$
(g) $\frac{1}{25}\sec^5(5x) - \frac{1}{15}\sec^3(5x) + C$
(h) $\frac{1}{2}x^2\arcsin x - \frac{1}{4}\arcsin x + \frac{1}{4}x\sqrt{1-x^2} + C$
(i) $-\frac{1}{2}x^4e^{-x^2} - x^2e^{-x^2} - e^{-x^2} + C$
(j) $\frac{1}{2}\sin(\ln x) - \frac{1}{2}x\cos(\ln x) + C$
(k) $\frac{1}{4}x^4\ln x - \frac{1}{16}x^4 + C$
(l) $-x\cot x + \ln|\sin x| + C$
(m) $\frac{2}{9}x^3(x^3+4)^{3/2} - \frac{4}{45}(x^3+4)^{5/2} + C$
(n) $x\sinh x - \cosh x + C$
(o) $\frac{1}{\ln 2}x2^x - \frac{1}{(\ln 2)^2}2^x + C$
(p) $\frac{2}{13}e^{2x}\sin(3x) - \frac{3}{13}e^{2x}\cos(3x) + C$
(q) $\left(1 - \frac{e^{4x}}{2}\right)\cos(e^{2x}) + e^{2x}\sin(e^{2x}) + C$
(r) $x\arccos x - \sqrt{1-x^2} + C$
(s) $\frac{2}{9}(e^{3/2}+2)$

54. $\frac{4}{3}$

55. $\pm\frac{\pi}{6} + \pi n$

56. The original expression is not an indeterminate form, so L'Hôpital's Rule does not apply. The correct value is 0.

57.

(a) 16π
(b) π
(c) $\frac{256\pi}{15}$
(d) $\frac{11\pi}{15}$
(e) $\frac{8\pi}{3}$
(f) $\frac{776\pi}{15}$

58. (a) $\frac{3\pi}{10}$
(b) $\frac{121\pi}{210}$
(c) $\frac{56\pi}{5}$
(d) $\frac{\pi(e-1)}{e}$

59. $11.25J$

60. 1.18×10^8 J

61. 2.94×10^6 J

62. 600 ft-lb

63. $17,640$ N

64. 19,500 lb

65. 14,560 lb

66. $g(x) = 2x + 1$, $c = 2$, $c = -3$

67. $G'(0) = 0$, $G'(0) = 0$, $G'(\pi/4) = 1$

68. (a) $\frac{25\pi}{4}$
(b) 4