Question 1:

(a)[5] Determine an equation of the tangent line to the curve

$$x^2 + y^2 = \frac{x}{y} + 1$$

at the point $(0, -1)$.

$$\frac{d}{dx}\left[x^2+y^2\right] = \frac{d}{dx}\left[\frac{x}{y}+1\right]$$

$$2x + 2yy' = \frac{y - xy'}{y^2}$$

at $(0,-1)$: $$0 + 2(-1)y' = \frac{-1-0}{(-1)^2}$$

$$\Rightarrow \quad y' = \frac{-1}{-2} = \frac{1}{2}$$

$$\therefore \quad y - (-1) = \frac{1}{2}(x - 0)$$

or $\boxed{y = \frac{1}{2}x - 1}$

(b)[5] Use logarithmic differentiation to determine y' where $y = (1+x)^{1/x}$.

$$\ln(y) = \ln\left[(1+x)^{\frac{1}{x}}\right]$$

$$\ln(y) = \frac{1}{x}\ln(1+x)$$

$$\frac{1}{y}y' = \left(\frac{-1}{x^2}\right)\ln(1+x) + \left(\frac{1}{x}\right)\left(\frac{1}{1+x}\right)$$

$$\boxed{y' = (1+x)^{\frac{1}{x}}\left[\left(\frac{-1}{x^2}\right)\ln(1+x) + \frac{1}{x(1+x)}\right]}$$

Question 2:

(a)[3] Differentiate $y = e^{\sin x} - \ln(\cos x)$

$$\boxed{y' = e^{\sin x} \cos x + \frac{\sin x}{\cos x}}$$

(b)[3] Differentiate $y = \log_7(x \tan x)$

$$\boxed{y' = \frac{1}{(x\tan x)\ln 7}\left[\tan x + x\sec^2 x\right]}$$

(c)[4] Determine $f''(0)$ where $f(x) = \ln(2 - e^x)$

$$f'(x) = \frac{1}{2-e^x}(-e^x) = \frac{e^x}{e^x-2}$$

$$f''(x) = \frac{(e^x-2)e^x - e^x(e^x)}{(e^x-2)^2}$$

$$f''(0) = \frac{(1-2)1 - 1(1)}{(1-2)^2} = \boxed{-2}$$

Question 3:

(a)[4] Determine $\lim_{x\to 3^+} \frac{x}{\ln(x-3)}$.

(If the limit does not exist but is ∞ or $-\infty$, state which with an explanation of your reasoning.)

As $x \to 3^+$, $x-3 \to 0^+$,

So $\ln(x-3) \to -\infty$,

so $\frac{x}{\ln(x-3)} \to 0$

$\therefore \lim_{x\to 3^+} \frac{x}{\ln(x-3)} = \boxed{0.}$

(b)[6] Use a linear approximation to estimate $\sqrt{16.1}$.

Here $f(x) = \sqrt{x} = x^{\frac{1}{2}}$, $a = 16$.

$f(a) = \sqrt{16} = 4$

$f'(x) = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

$f'(a) = \frac{1}{2\sqrt{16}} = \frac{1}{8}$,

$\therefore L(x) = f(a) + f'(a)(x-a)$

$L(x) = 4 + \frac{1}{8}(x-16).$

$\therefore \sqrt{16.1} = f(16.1) \approx L(16.1)$

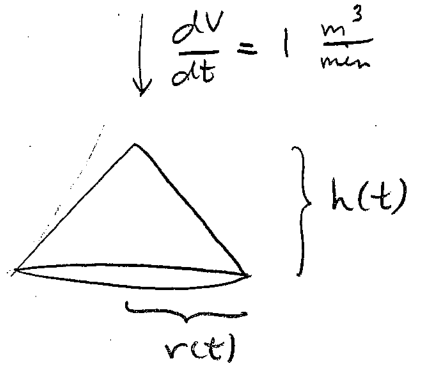
$= 4 + \frac{1}{8}(16.1 - 16)$

$= 4 + (\frac{1}{8})(\frac{1}{10})$

$= \boxed{\frac{321}{80}}$

Question 4 (related rates) [10]:

Sand is falling onto a pile at a rate of 1 m^3 per minute. The pile of sand maintains the shape of a cone with height equal to half the diameter of the base. How fast is the height of the sand pile increasing when the height is exactly 3 m? State units with your answer. (Recall: A cone of height h and base radius r has volume $V = \frac{\pi}{3} r^2 h$.)



$h = r$ at all times t.

Find $\frac{dh}{dt}$ when $h = 3$ m.

$$V = \frac{\pi}{3} r^2 h = \frac{\pi}{3} h^3$$

$$\frac{dV}{dt} = \frac{\pi}{3} 3h^2 \frac{dh}{dt}.$$

When $h = 3$ m:

$$1 = \frac{\pi}{\cancel{3}} \cancel{3} \cdot 3^2 \cdot \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{1}{9\pi} \frac{m}{min}$$

$\therefore$ height is increasing at $\frac{1}{9\pi} \frac{m}{min}$.

Question 5: For this question use the function $f(x) = x^2 e^x$

(a)[7] Determine the intervals of increase and decrease of $f(x)$. State a clear conclusion.

$f(x) = x^2 e^x$ has domain $(-\infty, \infty)$.

$f'(x) = 2xe^x + x^2 e^x = xe^x(2+x)$

- $f'(x) = 0$? $x = 0, -2$
- $f'(x)$ not exist? no such x in domain

		-2		0	
test values :	-3		-1		1
$f'(x) = e^x x(2+x)$:	$+$	0	$-$	0	$+$
$f(x) = x^2 e^x$:	$\nearrow$	$4e^{-2}$	$\searrow$	0	$\nearrow$

always positive

$\therefore$ f is increasing on $(-\infty, -2) \cup (0, \infty)$;

decreasing on $(-2, 0)$.

(b)[3] State the relative extrema of $f(x)$.

f has a rel. max. of $4e^{-2}$ at $x = -2$,

f has a rel. min. of 0 at $x = 0$.