Question 1:

(a)[5] Evaluate the following limit if it exists: $\lim_{\theta\to 0} \dfrac{\sin\theta}{\theta + \tan\theta}$

$$\lim_{\theta\to 0} \frac{\sin\theta}{\theta + \frac{\sin\theta}{\cos\theta}} \begin{matrix} \div \theta \\ \div \theta \end{matrix}$$

$$= \lim_{\theta\to 0} \frac{\left(\frac{\sin\theta}{\theta}\right)}{\frac{\theta}{\theta} + \frac{\sin\theta}{\theta}\cdot\frac{1}{\cos\theta}}$$

$$= \frac{1}{1+1\cdot 1}$$

$$= \boxed{\frac{1}{2}}$$

(b)[5] Determine $\lim_{x\to 0} x^4 \sin\left(\sqrt{1+x^2}\right)$.

(State any theorems used, like the the Squeeze Theorem, for example, and be sure to state the conditions necessary to justify use of the theorem.)

$$-x^4 \leq x^4 \sin\left(\sqrt{1+x^2}\right) \leq x^4$$

$$\lim_{x\to 0} -x^4 = 0 = \lim_{x\to 0} x^4$$

$\therefore$ by the Squeeze Theorem, $\lim_{x\to 0} x^4\sin\left(\sqrt{1+x^2}\right) = 0$

or, simply apply the direct substitution property:

$$\lim_{x\to 0} x^4 \sin\left(\sqrt{1+x^2}\right) = 0^4 \cdot \sin\left(\sqrt{1+0}\right) = 0.$$

Question 2:
Evaluate the following limits, if they exist:

(a)[4] $\lim_{x\to 0} \frac{\left[\frac{1}{x-1}+\frac{1}{x+1}\right]}{x}$ $= \lim_{x\to 0} \frac{\frac{x+1+x-1}{(x-1)(x+1)}}{\left(\frac{x}{1}\right)}$

$= \lim_{x\to 0} \frac{2\cancel{x}}{(x-1)(x+1)} \cdot \frac{1}{\cancel{x}}$

$= \boxed{-2}$

(b)[4] $\lim_{x\to 0^-} \left(\frac{1}{x}+\frac{1}{|x|}\right)$ $= \lim_{x\to 0^-} \frac{1}{x} - \frac{1}{x}$ since $x<0$

$= \boxed{0}$

(c)[2] $\lim_{x\to 2} \frac{\cos(\pi x)}{(x-2)^2}$ $\begin{matrix} \} \to \text{"}1\text{"} \\ \} \to 0 \end{matrix}$

$\boxed{\therefore \text{ limit does not exist.}}$

Question 3: Evaluate the following limits, if they exist:

(a)[3] $\lim_{x\to -3} \frac{x+3}{x^2+4x+3}$ $= \lim_{x\to -3} \frac{\cancel{(x+3)}}{\cancel{(x+3)}(x+1)}$

$= \boxed{\frac{-1}{2}}$

(b)[3] $\lim_{x\to 0} \frac{\sqrt{5x+4}-2}{x-2}$ $= \frac{0}{-2} = \boxed{0}$ (Direct Sub.)

(c)[4] $\lim_{x\to 1} \frac{x-1}{\sqrt{x+3}-2}$ $\cdot \frac{\sqrt{x+3}+2}{\sqrt{x+3}+2}$

$= \lim_{x\to 1} \frac{\cancel{(x-1)}(\sqrt{x+3}+2)}{\cancel{(x-1)}}$

$= \boxed{4}$

Question 4: For this question use the functions $f(x) = \sqrt{x+1}$, $g(x) = \frac{1}{x^2-5}$, $h(x) = \frac{1}{x}$.

(a)[3] Determine the domain of $(g \circ f)(x)$.

$$(g \circ f)(x) = g(f(x)) = \frac{1}{(\sqrt{x+1})^2 - 5} = \frac{1}{x-4}$$

Require : $x+1 \geq 0 \Rightarrow x \geq -1$

and $x \neq 4$

$\therefore$ domain is $[-1, 4) \cup (4, \infty)$.

(b)[3] Determine $(h \circ g \circ f)(x)$ and simplify.

$$(h \circ g \circ f)(x) = h(g(f(x))) = \frac{1}{\left[\frac{1}{(\sqrt{x+1})^2 - 5}\right]}$$

$$= \boxed{x-4}$$

(c)[2] Find a function $p(x)$ so that $g(x) = (h \circ p)(x)$.

Want $h(p(x)) = \frac{1}{x^2-5}$

$\therefore$ $\boxed{p(x) = x^2 - 5}$

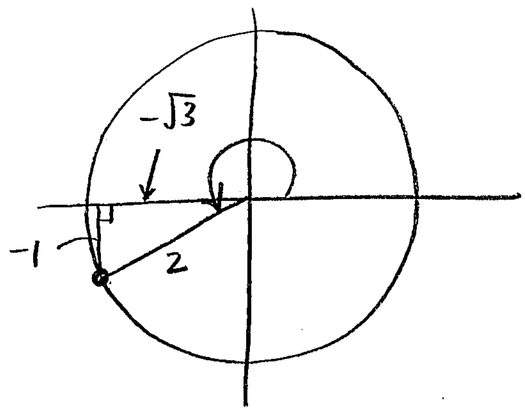
(d)[2] Find a function $q(x)$ so that $g(x) = (q \circ h)(x)$.

Want $q(h(x)) = \frac{1}{x^2-5}$

$\therefore$ $\boxed{q(x) = \frac{1}{(\frac{1}{x})^2 - 5}}$

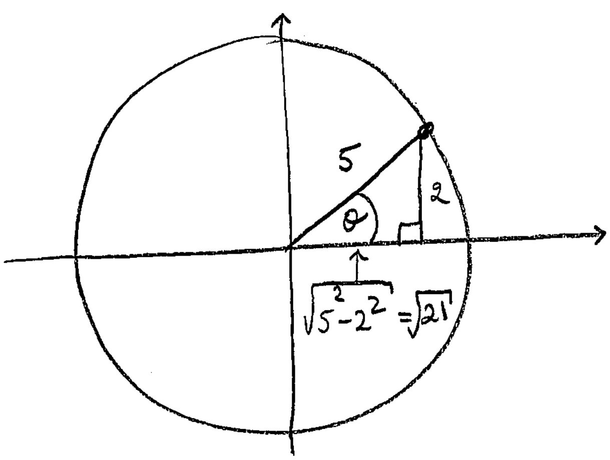
Question 5:

(a)[3] Determine the exact value of $\cos(7\pi/6)$.



$\therefore \cos\left(\frac{7\pi}{6}\right) = \boxed{\frac{-\sqrt{3}}{2}}$

(b)[3] Determine $\tan\theta$ if $\sin\theta = 2/5$ and $0 < \theta < \pi/2$.



$\therefore \tan\theta = \boxed{\frac{2}{\sqrt{21}}}.$

(c)[4] Find all values of x in $[0, 2\pi]$ that satisfy the equation $\sqrt{2}\cos x - 1 = 0$.

$\cos x = \frac{1}{\sqrt{2}}$:

$\boxed{\therefore \; x = \frac{\pi}{4}, \; \frac{7\pi}{4}.}$