(1) [5] Find the limit:

$$\lim_{x\to\infty} \sqrt{9x^2+x}-3x \qquad \sim \text{“}\infty-\infty\text{”}$$

$$= \lim_{x\to\infty} \frac{\sqrt{9x^2+x}-3x}{1}\left(\frac{\sqrt{9x^2+x}+3x}{\sqrt{9x^2+x}+3x}\right)$$

$$= \lim_{x\to\infty} \frac{\cancel{9x^2}+x-\cancel{9x^2}}{\sqrt{9x^2+x}+3x}$$

$$= \lim_{x\to\infty} \frac{x}{\sqrt{x^2\left(9+\frac{1}{x}\right)}+3x}$$

Note: $x>0$, so $\sqrt{x^2}=x$

$$= \lim_{x\to\infty} \frac{x}{x\sqrt{9+\frac{1}{x}}+3x}$$

$$= \lim_{x\to\infty} \frac{1}{\sqrt{9+\frac{1}{x}}+3}$$

$$= \boxed{\frac{1}{6}}$$

(2) [10] Find an equation of the tangent line to the curve $y = \sqrt{x}$ at the point $(1, 1)$. (Determine any required derivatives using the definition of the derivative.)

$$y' = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad \text{where } f(x) = \sqrt{x}$$

$$= \lim_{h \to 0} \frac{1}{h}\left(\sqrt{x+h} - \sqrt{x}\right) \cdot \left(\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}\right)$$

$$= \lim_{h \to 0} \frac{1}{h}\left(\frac{\cancel{x}+h-\cancel{x}}{\sqrt{x+h}+\sqrt{x}}\right)$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h}+\sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}}$$

$\therefore$ slope of tangent line is $y'\big|_{x=1} = \frac{1}{2\sqrt{1}} = \frac{1}{2}$

$\therefore$ Equation of tangent line is $\boxed{y - 1 = \frac{1}{2}(x-1)}$