

# 1 Exponentials

**General Base**  $a > 0$

**Special Case: Base**  $e = 2.71828\dots$

**Laws:**

$$a^b a^c = a^{b+c}$$

$$e^b e^c = e^{b+c}$$

$$\frac{a^b}{a^c} = a^{b-c}$$

$$\frac{e^b}{e^c} = e^{b-c}$$

$$(a^b)^c = a^{bc}$$

$$(e^b)^c = e^{bc}$$

**Derivative:**

$$\frac{d}{dx} [a^x] = a^x \ln(a)$$

$$\frac{d}{dx} [e^x] = e^x$$

# 2 Logarithms

**Definiton:**  $\log_a(b)$  is the power to which  $a$  is raised to give  $b$ .

**Definiton:**  $\ln(b) = \log_e(b)$ , the power to which  $e$  is raised to give  $b$ .

**General Base**  $a > 0$

**Special Case: Base**  $e = 2.71828\dots$

**Laws:**

$$\log_a(bc) = \log_a(b) + \log_a(c)$$

$$\ln(bc) = \ln(b) + \ln(c)$$

$$\log_a\left(\frac{b}{c}\right) = \log_a(b) - \log_a(c)$$

$$\ln\left(\frac{b}{c}\right) = \ln(b) - \ln(c)$$

$$\log_a(b^c) = c \log_a(b)$$

$$\ln(b^c) = c \ln(b)$$

**Change of Base:**

$$\log_b(c) = \frac{\log_a(c)}{\log_a(b)}$$

$$\log_b(c) = \frac{\ln(c)}{\ln(b)}$$

**Derivative:**

$$\frac{d}{dx} [\log_a(x)] = \frac{1}{x \ln(a)}$$

$$\frac{d}{dx} [\ln(x)] = \frac{1}{x}$$

# 3 Inverse Properties

**General Base**  $a > 0$

**Special Case: Base**  $e = 2.71828\dots$

$$a^{\log_a(x)} = x$$

$$e^{\ln(x)} = x$$

$$\log_a(a^x) = x$$

$$\ln(e^x) = x$$