Question 1: Evaluate the following limits, showing all work. If a limit does not exist but is ∞ or $-\infty$, state which, with an explanation of your reasoning.

(a)[2] $\lim_{x \to 7^-} \dfrac{7x}{x-7}$

(b)[2] $\lim_{t \to -\infty} \dfrac{t^3 - t + 1}{t^4 + 3t^2 - 2}$

(c)[3] $\lim_{x \to \infty} x^2 - x^4$

(d)[3] $\lim_{x \to \infty} \dfrac{x+2}{\sqrt{9x^2+1}}$

Question 2: Suppose $h(t) = 20t - kt^2$ describes the height in metres above the ground of a projectile launched with an initial velocity of 20 m/s, where time $t \geq 0$. Here k is a positive constant, and note that some of your answers may contain this constant k. State units with your answers.

(a)[2] At what time(s) is the object at ground level?

(b)[3] At what time in the projectile's flight will it begin falling back to earth?

(c)[2] What is the acceleration of the projectile at time $t = 1$ second?

(d)[3] What is the velocity of the projectile when it hits the ground?

Question 3: Determine the derivatives of the following functions. It is not necessary to simplify your final answers.

(a)[3] $y = \dfrac{x^3}{3} - 3\sqrt[3]{x} + \sqrt{3}$

(b)[4] $f(x) = (x - \cos x)\left(\dfrac{4}{x} - \sin x\right)$

(c)[3] $g(x) = \dfrac{\tan(2x)}{\pi^2 + x^2}$

Question 4: Determine the derivatives of the following functions. It is not necessary to simplify your final answers.

(a)[3] $y = \sec\left(\sqrt{x} - x^5\right)$

(b)[3] $f(\theta) = \sqrt{3\theta - \theta \sin\theta}$

(c)[4] $g(t) = \left[t + \cos\left(\frac{1}{\sqrt{t}}\right)\right]^{121}$

Question 5:

(a)[5] Determine the equation of the tangent line to the curve $x^3 - 5xy^2 + y^3 = xy - 3$ at the point $(2, 1)$.

(b)[5] There are two tangent lines to the curve $y = x^2 + x$ which pass through the point $(2, -3)$. Determine the points at which these tangent lines contact the curve.