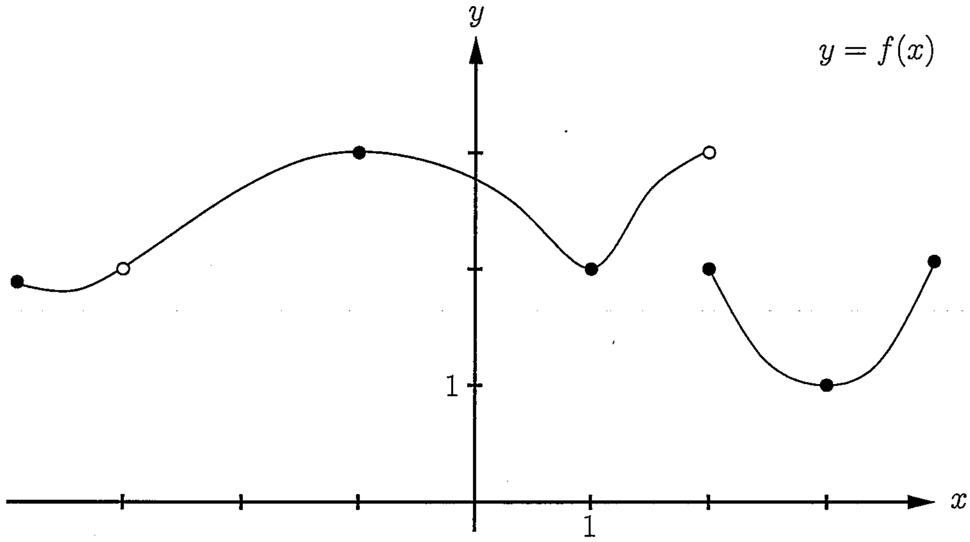
Question 1: For this question use the graph of $y = f(x)$ below:



(a)[2 points] What is the range of $f(x)$?

$[1, 3]$

(b)[2 points] What is $(f \circ f)(-1)$?

$(f \circ f)(-1) = f(f(-1)) = f(3) = 1$

(c)[2 points] What is $\lim_{x \to -3} f(x)$?

$\lim_{x \to -3} f(x) = 2$

(d)[2 points] What is $\lim_{x \to 2^-} f(x)$?

$\lim_{x \to 2^-} f(x) = 3$

(e)[2 points] Is f continuous at $x = -3$? Explain your answer using limits.

No. $\lim_{x \to -3} f(x) = 2$, but $f(-3)$ is not defined,

so $\lim_{x \to -3} f(x) \neq f(-3)$.

Question 2:

(a)[5 points] Evaluate: $\lim_{x\to -5} \frac{x^2+3x-10}{x^2+8x+15}$ $\}\to 0$ $\}\to 0$

$$\lim_{x\to -5} \frac{x^2+3x-10}{x^2+8x+15} = \lim_{x\to -5} \frac{\cancel{(x+5)}(x-2)}{\cancel{(x+5)}(x+3)}$$

$$= \frac{-7}{-2}$$

$$= \boxed{\frac{7}{2}}$$

(b)[3 points] Evaluate: $\lim_{x\to \pi} \frac{3x^3 - 2\sin^2(2x)}{x\cos(x)}$

$$\lim_{x\to \pi} \frac{3x^3 - 2\sin^2(2x)}{x\cos(x)} = \frac{3\pi^3 - \cancel{2\sin^2(2\pi)}^{\to 0}}{\pi \cancel{\cos(\pi)}^{\to -1}}$$

$$= \frac{3\pi^3}{-\pi}$$

$$= \boxed{-3\pi^2}$$

(c)[2 points] Evaluate: $\lim_{x\to -6} \frac{\sqrt{10+x}}{x^2-36}$ $\}\to$ "$\frac{2}{0}$"

$\therefore \lim_{x\to -6} \frac{\sqrt{10+x}}{x^2-36}$ does not exist.

Question 3:

(a)[5 points] Evaluate: $\lim_{x\to 2} \frac{\sqrt{11-x}-3}{x-2}$ $\begin{matrix} \} \to 0 \\ \} \to 0 \end{matrix}$

$$\lim_{x\to 2} \frac{\sqrt{11-x}-3}{x-2} \cdot \frac{\sqrt{11-x}+3}{\sqrt{11-x}+3} = \lim_{x\to 2} \frac{11-x-9}{(x-2)(\sqrt{11-x}+3)}$$

$$= \lim_{x\to 2} \frac{-\cancel{(x-2)}}{\cancel{(x-2)}(\sqrt{11-x}+3)}$$

$$= \boxed{\frac{-1}{6}}$$

(b)[3 points] Evaluate: $\lim_{x\to -4^+} \frac{|x+4|}{3x+12}$ $\begin{matrix} \} \to 0 \\ \} \to 0 \end{matrix}$

as $x \to -4^+$, $x > -4$, so $x+4 > 0$, so $|x+4| = x+4$.

$$\therefore \lim_{x\to -4^+} \frac{|x+4|}{3x+12} = \lim_{x\to -4^+} \frac{\cancel{(x+4)}}{3\cancel{(x+4)}} = \boxed{\frac{1}{3}}$$

(c)[2 points] Evaluate: $\lim_{x\to\pi} \frac{(x-\pi)^2}{\pi x}$ $\begin{matrix} \} \to 0 \\ \} \to \pi^2 \end{matrix}$

$$\therefore \lim_{x\to\pi} \frac{(x-\pi)^2}{\pi x} = \boxed{0}$$

Question 4:

(a)[5 points] Evaluate: $\lim_{\theta\to 0} \dfrac{\sin(2\theta)+\sin(3\theta)}{3\theta+\tan(\theta)}$ $\}\to 0$ $\}\to 0$

$$\lim_{\theta\to 0}\frac{\sin(2\theta)+\sin(3\theta)}{3\theta+\tan(\theta)} = \lim_{\theta\to 0}\frac{2\cdot\frac{\sin(2\theta)}{2\theta}+3\cdot\frac{\sin(3\theta)}{3\theta}}{\frac{3\theta}{\theta}+\frac{\sin\theta}{\theta}\cdot\frac{1}{\cos\theta}}$$

$$=\frac{2+3}{3+1}$$

$$=\boxed{\frac{5}{4}}$$

(b)[5 points] Evaluate: $\lim_{x\to 0}\left(\dfrac{x^2}{1+x}\right)\cos\left(\dfrac{1}{x}\right)$ using the Squeeze Theorem. Carefully show all steps to support your argument.

$$-1\le\cos\left(\tfrac{1}{x}\right)\le 1$$

$$\therefore\quad -\frac{x^2}{1+x}\le\left(\frac{x^2}{1+x}\right)\cos\left(\frac{1}{x}\right)\le\frac{x^2}{1+x}$$

since $\lim_{x\to 0}\left(-\dfrac{x^2}{1+x}\right)=0=\lim_{x\to 0}\dfrac{x^2}{1+x}$,

by the Squeeze Theorem,

$$\lim_{x\to 0}\left(\frac{x^2}{1+x}\right)\cos\left(\frac{1}{x}\right)=0.$$

Question 5:

(a)[4 points] Let $f(x) = \dfrac{1}{1-x}$ and $g(x) = \dfrac{1+x}{x}$. Determine and simplify $(f \circ g)(x)$ and determine its domain.

$$(f \circ g)(x) = \frac{1}{1 - \left(\frac{1+x}{x}\right)} = \frac{x}{x - (1+x)} = \frac{x}{-1} = -x$$

Using $\nearrow$, must have $x \neq 0$, $\frac{1+x}{x} \neq 1$.

If $\frac{1+x}{x} = 1$, $1 + x = x$, so $1 = 0$: impossible.

$\therefore$ only restriction on domain is $x \neq 0$, so domain of $f \circ g$ is $(-\infty, 0) \cup (0, \infty)$.

(b)[4 points] Let $H(x) = \left(1 + \sqrt{1 + \sin x}\right)^3$ and $g(x) = \sqrt{1+x}$. Find functions f and h so that $H = f \circ g \circ h$.

$h(x) = \sin x$

$g(x) = \sqrt{1 + x}$

$f(x) = (1 + x)^3$.

(c)[2 points] Give an example of functions $f(x)$ and $g(x)$ such that neither $\lim_{x \to 0} f(x)$ nor $\lim_{x\to 0} g(x)$ exist, yet $\lim_{x \to 0}[f(x) - g(x)]$ does exist.

$f(x) = \frac{1}{x}$, $g(x) = \frac{1}{x}$

$\lim_{x \to 0} \frac{1}{x}$ does not exist, yet

$\lim_{x\to 0} \left(\frac{1}{x} - \frac{1}{x}\right) = \lim_{x \to 0} 0 = 0$.