Math 121 M10N01 - Quiz 3 Name: SOLUTIONS.
June 3 2010 Stu#:

(1) [3] Evaluate the following limit. If the limit does not exist but is ∞ or $-\infty$, state which, with an explanation of your reasoning.

$$\lim_{x\to 3^+} e^{2/(3-x)}$$

As $x \to 3^+$, $3-x \to 0^-$, so $\frac{2}{3-x} \to -\infty$, so $e^{2/(3-x)} \to 0$

$$\therefore \lim_{x\to 3^+} e^{\frac{2}{3-x}} = \boxed{0}$$

(2) [4] Differentiate: $y = \ln\left(e^{-x} + xe^{-x}\right)$

$$y' = \frac{1}{e^{-x}+xe^{-x}}\left[-e^{-x}+e^{-x}-xe^{-x}\right]$$

$$= \frac{-x\cancel{e^{-x}}}{\cancel{e^{-x}}(1+x)}$$

$$= \frac{-x}{1+x}$$

or:

$$y = \ln\left[e^{-x}(1+x)\right]$$

$$= \ln(e^{-x}) + \ln(1+x)$$

$$= -x + \ln(1+x)$$

$$\therefore y' = -1 + \frac{1}{1+x}.$$

(3) [3] Differentiate: $y = 2^{\cos(\pi x)}$

$$y' = 2^{\cos(\pi x)} \cdot \ln(2)\left(-\sin(\pi x)\right)(\pi)$$

(4) [5] Use logarithmic differentiation to find the derivative of $y = (\tan x)^{1/x}$

$$y = (\tan x)^{1/x}$$

$$\ln y = \frac{1}{x}\ln(\tan x)$$

$$\frac{1}{y}y' = -x^{-2}\ln(\tan x) + \frac{1}{x}\frac{1}{\tan x}\cdot\sec^2 x$$

$$\therefore y' = (\tan x)^{\frac{1}{x}}\left[\frac{-\ln(\tan x)}{x^2} + \frac{\sec^2 x}{x\tan x}\right].$$

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(5) [5 bonus points] There are two tangent lines to the curve $y = x^2 - x$ which pass through the point $(-2, -3)$. Determine the points at which these tangent lines contact the curve.

Let (a, a^2-a) be the point of contact between the tangent line and curve $y = x^2 - x$.

$$\therefore \quad m = \frac{a^2-a-(-3)}{a-(-2)} = \frac{a^2-a+3}{a+2}$$

but also:

$$m = \frac{d}{dx}\left[x^2-x\right]\Big|_{x=a} = 2x-1\Big|_{x=a} = 2a-1$$

$$\therefore \quad \frac{a^2-a+3}{a+2} = 2a-1$$

$$a^2-a+3 = (2a-1)(a+2)$$

$$a^2-a+3 = 2a^2+3a-2$$

$$a^2+4a-5=0$$

$$(a-1)(a+5)=0$$

$$\therefore \quad a=1 \quad , \quad a=-5$$

$$a^2-a = 0 \quad , \quad a^2-a = 30$$

$\therefore$ points are $(1,0)$ & $(-5,30)$