Math 121 M10N01 - Quiz 2 Name: SOLUTIONS.
May 20 2010 Stu#:

Evaluate the following limits. If a limit does not exist but is ∞ or $-\infty$, state which, with an explanation of your reasoning.

(1) [3 points] $\lim\limits_{x\to 3} \dfrac{2-x^2}{(x-3)^2}$ As $x \to 3$, $2-x^2 \to -7$ and $(x-3)^2 \to 0^+$.

$$\therefore \left.\begin{aligned} \lim_{x\to 3^+} \frac{2-x^2}{(x-3)^2} &= -\infty \\ \lim_{x\to 3^-} \frac{2-x^2}{(x-3)^2} &= -\infty \end{aligned}\right\} \quad \therefore \lim_{x\to 3} \frac{2-x^2}{(x-3)^2} = \boxed{-\infty.}$$

(2) [3 points] $\lim\limits_{u\to\infty} \dfrac{4u^4+5}{(u^2-2)(2u^2-1)}$

$$= \lim_{u\to\infty} \frac{4u^4+5}{2u^4-5u^2+2}$$

$$= \lim_{u\to\infty} \frac{\cancel{u^4}\left(4+\frac{5}{u^4}\right)}{\cancel{u^4}\left(2-\frac{5}{u^2}+\frac{2}{u^4}\right)}$$

$$= \boxed{2}$$

(3) [4 points] $\lim\limits_{x\to\infty} \dfrac{\sqrt{16x^2+2x}-4x}{1} \cdot \dfrac{\sqrt{16x^2+2x}+4x}{\sqrt{16x^2+2x}+4x}$

$$= \lim_{x\to\infty} \frac{16x^2+2x-16x^2}{\sqrt{x^2\left(16+\frac{2}{x}\right)}+4x}$$

$$= \lim_{x\to\infty} \frac{2\cancel{x}}{\cancel{x}\sqrt{16+\frac{2}{x}}+4\cancel{x}}$$

$$= \frac{2}{8}$$

$$= \boxed{\frac{1}{4}}$$

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(4) [5 points] Use the definition of the derivative (not the derivative rules) to find an equation of the tangent line to the graph of $y = \dfrac{1}{\sqrt{x+3}}$ at the point where $x = 1$.

Here $f(x) = \dfrac{1}{\sqrt{x+3}}$; point on tangent line is

$$(1, f(1)) = \left(1, \frac{1}{\sqrt{1+3}}\right) = \left(1, \frac{1}{2}\right).$$

Slope of tangent line is

$$f'(1) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{1}{\sqrt{1+h+3}} - \frac{1}{\sqrt{1+3}}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{1}{\sqrt{4+h}} - \frac{1}{2}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{2-\sqrt{4+h}}{2\sqrt{4+h}} \cdot \frac{2+\sqrt{4+h}}{2+\sqrt{4+h}}\right]$$

$$= \lim_{h\to 0} \frac{1}{\cancel{h}}\left[\frac{\cancel{4}-\cancel{4}-\cancel{h}}{2\sqrt{4+h}\,(2+\sqrt{4+h})}\right]$$

$$= \frac{-1}{2\sqrt{4}\,(2+\sqrt{4})}$$

$$= \frac{-1}{16}.$$

$\therefore$ Equation of tangent line is

$$y - \frac{1}{2} = \frac{-1}{16}(x-1).$$