Question 1:

(a)[**3 points**] Evaluate $\lim_{x\to\infty} \frac{x^3-2x+3}{5-2x^3}$ $\begin{matrix} \div x^3 \\ \div x^3 \end{matrix}$

$$= \lim_{x\to\infty} \frac{1 - \frac{2}{x^2} + \frac{3}{x^3}}{\frac{5}{x^3} - 2}$$

$$= \boxed{\frac{-1}{2}}$$

(b)[**3 points**] Determine u': $u = \sqrt[4]{t^3} + 2\sqrt[3]{t^2}$ $= t^{\frac{3}{4}} + 2t^{2/3}$

$$\therefore \quad u' = \frac{3}{4}t^{-\frac{1}{4}} + \frac{4}{3}t^{-\frac{1}{3}}$$

(c)[**4 points**] A ball with an initial velocity of 5 m/s rolls down a hill. The position of the ball after t seconds is $s(t) = 5t + 3t^2$ metres. How long does it take the velocity to reach 35 m/s?

$$v(t) = s'(t) = 5 + 6t$$

$$5 + 6t = 35$$

$$\Rightarrow \quad 6t = 30$$

$$\Rightarrow \quad \boxed{t = 5 \text{ s.}}$$

Question 2:

(a)[3 points] Determine $\frac{dy}{dx}$: $y = \frac{1+\sin(x)}{x^2 - \cos(x)}$

$$\frac{dy}{dx} = \frac{[x^2 - \cos(x)]\cos(x) - [1+\sin(x)][2x+\sin(x)]}{[x^2-\cos(x)]^2}$$

(b)[3 points] Determine $f'(x)$: $f(x) = (\sqrt{x} + 3x^2)\tan(x)$ $= (x^{\frac{1}{2}} + 3x^2)\tan(x)$

$$f'(x) = \left(\tfrac{1}{2}x^{-\frac{1}{2}} + 6x\right)\tan(x) + (x^{1/2} + 3x^2)\sec^2(x)$$

(c)[4 points] Determine $g''(0)$: $g(\theta) = \sec(\theta)$

$$g'(\theta) = \sec(\theta)\tan(\theta)$$

$$g''(\theta) = \sec(\theta)\tan(\theta)\tan(\theta) + \sec(\theta)\sec^2(\theta)$$

$$g''(0) = \cancel{\sec(0)\tan(0)}\tan(0) + \sec(0)\sec^2(0)$$

$0 \quad 1 \quad 1$

$$= \boxed{1}$$

Question 3:

(a)[3 points] Determine $\frac{dy}{dx}$: $y = (x^2+1)\sqrt[3]{x^2+2} = (x^2+1)(x^2+2)^{\frac{1}{3}}$

$$\frac{dy}{dx} = 2x(x^2+2)^{1/3} + (x^2+1)\frac{1}{3}(x^2+2)^{-\frac{2}{3}}(2x)$$

(b)[3 points] Determine $f'(x)$: $f(x) = \frac{x}{\sqrt{7-3x}} = x(7-3x)^{-\frac{1}{2}}$

$$f'(x) = (7-3x)^{-\frac{1}{2}} + x\left(-\tfrac{1}{2}\right)(7-3x)^{-\frac{3}{2}}(-3)$$

(c)[4 points] Determine y': $y = \sin\left(\tan\sqrt{\sin(x)}\right) = \sin\left(\tan\left([\sin(x)]^{1/2}\right)\right)$

$$y' = \cos\left(\tan\left([\sin(x)]^{\frac{1}{2}}\right)\right)\cdot\sec^2\left([\sin(x)]^{\frac{1}{2}}\right)\cdot\frac{1}{2}[\sin(x)]^{-\frac{1}{2}}\cdot\cos(x)$$

Question 4:

(a) [5 points] Find an equation of the tangent line to $y = \sqrt{1 + 4\sin(x)}$ at the point where $x = 0$.

At $x=0$, $y=\sqrt{1+4\sin(0)}=1$, so $(0,1)$ is a point on the line.

Slope of line is $y'\Big|_{x=0} = \frac{1}{2}(1+4\sin(x))^{-\frac{1}{2}}(4\cos(x))\Big|_{x=0}$

$= 2(1+0)^{-\frac{1}{2}} \cdot 1$

$= 2$

$\therefore$ Equation is $y-1 = 2(x-0)$

or $\boxed{y = 2x+1}$

(b) [5 points] Determine y' using implicit differentiation: $1 + x = \sin(xy^2)$

$$\frac{d}{dx}[1+x] = \frac{d}{dx}[\sin(xy^2)]$$

$$1 = \cos(xy^2)(y^2 + x \cdot 2y\,y')$$

$$1 = y^2\cos(xy^2) + 2xy\cos(xy^2)\,y'$$

$$\therefore \boxed{y' = \frac{1 - y^2\cos(xy^2)}{2xy\cos(xy^2)}}$$

Question 5:

(a)[**5 points**] Suppose $h(x) = f(x)g(x) + [g(x)]^2$. If $f(2) = 3$, $f'(2) = -2$, $g(2) = 5$ and $g'(2) = 4$, calculate $h'(2)$.

$$h'(x) = f'(x)g(x) + f(x)g'(x) + 2[g(x)] \cdot g'(x)$$

$$\therefore h'(2) = f'(2)g(2) + f(2)g'(2) + 2g(2)g'(2)$$

$$= (-2)(5) + (3)(4) + 2(5)(4)$$

$$= \boxed{42}$$

(b)[**5 points**] Find all values of t at which tangent lines to the curve $y = \dfrac{t^2}{1+t}$ are horizontal.

$$y' = \frac{(1+t)(2t) - t^2}{(1+t)^2} = \frac{2t + 2t^2 - t^2}{(1+t)^2}$$

$$= \frac{t^2 + 2t}{(1+t)^2}$$

$$y' = 0 \Rightarrow t(t+2) = 0$$

$$\Rightarrow \boxed{t = 0, \quad t = -2}$$