Question 1:

(a) [4 **points**] Let $f(x) = \frac{1}{\sqrt{x+2}}$ and $g(x) = \frac{1}{x^2}$. Find and simplify $(g \circ f)(x)$ and state the domain.

$$(g \circ f)(x) = g(f(x)) = \frac{1}{\left(\frac{1}{\sqrt{x+2}}\right)^2} = \frac{1}{\left(\frac{1}{x+2}\right)} = x+2 .$$

Using , must have $x+2>0$, so $x>-2$.

$\therefore$ $(g \circ f)(x) = x+2$, domain $(-2, \infty)$.

(b) [4 **points**] Let $f(x) = \frac{1}{x}$. Evaluate and simplify $\frac{f(1+h)-f(1)}{h}$.

$$\frac{f(1+h)-f(1)}{h} = \frac{\left(\frac{1}{1+h} - \frac{1}{1}\right)}{h}$$

$$= \frac{1}{h}\left[\frac{1-(1+h)}{1+h}\right]$$

$$= \frac{1}{h} \frac{-h}{1+h}$$

$$= \frac{-1}{1+h}$$

(c) [2 **points**] Let $H(x) = 5\sqrt{\tan x} - \tan^5 x$. Find functions f and g such that $H = f \circ g$.

$g(x) = \tan(x)$

$f(x) = 5\sqrt{x} - x^5$

Question 2:

(a)[5 points] Evaluate $\displaystyle\lim_{x\to 3}\frac{\sqrt{x+22}-5}{x-3}$.

$$= \lim_{x\to 3}\frac{\sqrt{x+22}-5}{x-3}\cdot\frac{\sqrt{x+22}+5}{\sqrt{x+22}+5}$$

$$= \lim_{x\to 3}\frac{x+22-25}{(x-3)(\sqrt{x+22}+5)}$$

$$= \lim_{x\to 3}\frac{\cancel{(x-3)}}{\cancel{(x-3)}(\sqrt{x+22}+5)}$$

$$= \boxed{\frac{1}{10}}$$

(b)[5 points] Evaluate $\displaystyle\lim_{t\to 4}\frac{t^2-t-12}{t^2-6t+8}$.

$$= \lim_{t\to 4}\frac{\cancel{(t-4)}(t+3)}{\cancel{(t-4)}(t-2)}$$

$$= \boxed{\frac{7}{2}}$$

Question 3:

(a)[5 points] Evaluate $\lim_{x\to -2} \frac{\left(\frac{1}{4x}+\frac{1}{8}\right)}{x+2}$.

$$= \lim_{x\to -2} \frac{1}{\cancel{x+2}}\left[\frac{\cancel{2+x}}{8x}\right]$$

$$= \boxed{\frac{-1}{16}}$$

(b)[5 points] Evaluate $\lim_{x\to -2^-} \frac{x^2-2}{x-2}$.

$$\lim_{x\to -2^-} \frac{x^2-2}{x-2}$$

$$= \frac{(-2)^2-2}{-2-2}$$

$$= \frac{2}{-4}$$

$$= \boxed{\frac{-1}{2}}$$

Question 4:

(a) [5 points] Evaluate $\lim_{x\to 0} \frac{\sin(2x)}{6x - \sin(3x)}$ $\begin{matrix} \div x \\ \div x \end{matrix}$

$$= \lim_{x\to 0} \frac{2 \cdot \frac{\sin(2x)}{2x}}{\frac{6\cancel{x}}{\cancel{x}} - 3 \cdot \frac{\sin(3x)}{3x}}$$

$$= \frac{2 \cdot 1}{6 - 3 \cdot 1}$$

$$= \boxed{\frac{2}{3}}$$

(b) [5 points] Use the Intermediate Value Theorem to show that the equation $\sqrt{\frac{x}{\pi}} = \cos\left(\frac{x}{2}\right)$ has a solution on the interval $[0, \pi]$.

Show $\sqrt{\frac{x}{\pi}} - \cos\left(\frac{x}{2}\right) = 0$ has solution on $[0, \pi]$.

Let $f(x) = \sqrt{\frac{x}{\pi}} - \cos\left(\frac{x}{2}\right)$. f is continuous on $[0, \pi]$.

$f(0) = -1$, $f(\pi) = 1$, and $f(0) < 0 < f(\pi)$.

$\therefore$ By I.V.T., $f(c) = 0$ for some $0 < c < \pi$.

i.e. $\sqrt{\frac{c}{\pi}} - \cos\left(\frac{c}{2}\right) = 0$ for some $0 < c < \pi$.

Question 5:

(a)[5 points] Determine $\lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right)$. The Squeeze Theorem may help here.

$$-x^2 \leq x^2 \sin\left(\frac{1}{x}\right) \leq x^2$$

Since $\lim_{x\to 0} -x^2 = 0 = \lim_{x\to 0} x^2$,

by the Squeeze Theorem, $\lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right) = 0$.

(b)[5 points] Determine if the function

$$f(x) = \begin{cases} \dfrac{1+x^3}{\sqrt{x+3}}, & \text{if } x \geq 1 \\ 4x^{17} - x - 2, & \text{if } x < 1 \end{cases}$$

is continuous at $x = 1$.

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} 4x^{17} - x - 2 = 4 - 1 - 2 = 1$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \frac{1+x^3}{\sqrt{x+3}} = \frac{1+1}{\sqrt{1+3}} = 1$$

$$f(1) = \frac{1+1^3}{\sqrt{1+3}} = 1$$

$\therefore$ Since $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1)$, f is continuous at $x = 1$.