Math 121 M09N01 - Quiz 3
June 8 2009

Name: SOLUTIONS.
Stu#:

(1) [2 points] Find the derivative of $y = e^{x\cos x}$.

$$y' = e^{x\cos x}\left[\cos x - x\sin x\right]$$

(2) [4 points] Find an equation of the tangent line to $y = \ln(\ln x)$ at the point $(e, 0)$.

$$\frac{dy}{dx} = \frac{1}{\ln x}\cdot\frac{1}{x}$$

$$\therefore \left.\frac{dy}{dx}\right|_{x=e} = \frac{1}{\cancel{\ln e}\ 1}\cdot\frac{1}{e} = \frac{1}{e}$$

$\therefore$ Equation is $y - 0 = \frac{1}{e}(x - e)$

or $y = \frac{1}{e}\cdot x - 1$

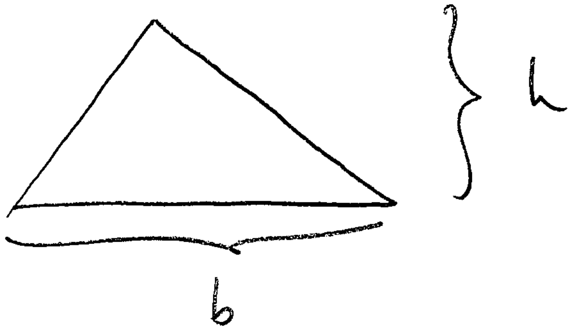
(3) [4 points] Use logarithmic differentiation to find the derivative of $y = (\tan x)^{1/x}$.

$$\ln y = \frac{1}{x}\ln(\tan x)$$

$$\frac{1}{y}y' = \frac{-1}{x^2}\ln(\tan x) + \frac{1}{x}\,\frac{1}{\tan x}\cdot\sec^2 x$$

$$\therefore y' = (\tan x)^{\frac{1}{x}}\left[\frac{-1}{x^2}\ln(\tan x) + \frac{1}{x}\,\frac{\sec^2 x}{\tan x}\right].$$

(4) [5 points] The altitude of a triangle is increasing at a rate of 1 cm/min while the area of the triangle is increasing at a rate of 2 cm^2/min. At what rate is the base of the triangle changing when the altitude is 10 cm and the area is 100 cm^2?



$$\frac{dh}{dt} = 1 \frac{cm}{min}$$

$$\frac{dA}{dt} = 2 \frac{cm^2}{min}$$

Find $\frac{db}{dt}$ when $h = 10$ cm & $A = 100$ cm^2.

$$A = \frac{1}{2} bh$$

$$\therefore \quad b = \frac{2A}{h}$$

$$\frac{db}{dt} = 2\left[\frac{hA' - Ah'}{h^2}\right]$$

$$\therefore \left.\frac{db}{dt}\right|_{\substack{h=10 \\ A=100}} = 2\left[\frac{(10)(2) - (100)(1)}{10^2}\right]$$

$$= -\frac{160}{100}$$

$$= -\frac{8}{5} \frac{cm}{min}$$

$\therefore$ The base is decreasing at $-\frac{8}{5} \frac{cm}{min}$.