

(1) [6 points] Let $f(x) = \frac{1}{\sqrt{x+2}}$. Use the limit definition of the derivative to find $f'(a)$.

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{1}{\sqrt{a+h+2}} - \frac{1}{\sqrt{a+2}} \right]$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{\sqrt{a+2} - \sqrt{a+h+2}}{\sqrt{a+h+2} \sqrt{a+2}} \cdot \frac{\sqrt{a+2} + \sqrt{a+h+2}}{\sqrt{a+2} + \sqrt{a+h+2}} \right]$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{\cancel{a+2} - \cancel{a} - h - 2}{\sqrt{a+h+2} \sqrt{a+2} (\sqrt{a+2} + \sqrt{a+h+2})} \right]$$

$$= \lim_{h \rightarrow 0} \frac{-1}{\sqrt{a+h+2} \sqrt{a+2} (\sqrt{a+2} + \sqrt{a+h+2})}$$

$$= \frac{-1}{(a+2) 2\sqrt{a+2}}$$

$$= \frac{-1}{2(a+2)^{3/2}}$$

$$= -\frac{1}{2} (a+2)^{-3/2}$$

(2) [3 points] Differentiate $v = t^2 - \frac{1}{\sqrt[4]{t^3}} = t^2 - t^{-\frac{3}{4}}$

$$\frac{dv}{dt} = 2t + \frac{3}{4} t^{-\frac{7}{4}}$$

(3) [3 points] Differentiate $h(\theta) = \theta \csc \theta - \cot \theta$.

$$\begin{aligned} h'(\theta) &= 1 \cdot \csc \theta + \theta (-\csc \theta \cot \theta) - (-\csc^2 \theta) \\ &= \csc \theta - \theta \csc \theta \cot \theta + \csc^2 \theta \end{aligned}$$

(4) [3 points] Differentiate $y = \frac{\sin x}{x^2}$.

$$\begin{aligned} \frac{dy}{dx} &= \frac{x^2 \cos x - \sin x (2x)}{(x^2)^2} \\ &= \frac{x^2 \cos x - 2x \sin x}{x^4} \end{aligned}$$