Question 1 [15 points]: Differentiate the following functions (you do not have to simplify your answers, however points will be deducted for improper use of notation):

(a) [**3**] $y = 2x^3 + \dfrac{3}{x}$

(b) [**3**] $f(x) = (x^2 + x + 1)\sqrt{x^2 + 2}$

(c) [**3**] $g(t) = \sec\left(\dfrac{1}{t^2}\right)$

(d) [**3**] $y = \dfrac{4x^3}{\tan x}$

(e) [**3**] $f(x) = 5^x x^5$

Question 2 [14 points]:

(a) [**3**] Find $\dfrac{dy}{dx}$ (you do not have to simplify your answer): $y = \left(\sin^2 x\right) \ln\left(1 - x^2\right)$

(b) [**3**] Find and simplify $f'(\pi/2)$ where $f(x) = e^{\cos x}$.

(c) [**4**] Compute $g''(2)$ if $g(t) = \ln\left(1 + \sin\left(\pi t\right)\right)$

(d) [**4**] Find the derivative (you do not have to simplify your answer): $y = \sqrt[3]{e^{2x} x^3}$

Question 3 [12 points]: Evaluate the following limits (it may be useful to recall that $\lim_{x\to 0} \frac{\sin x}{x} = 1$):

(a) [3] $\lim_{t\to 1} \frac{t^2-1}{t^2-3t+2}$

(b) [3] $\lim_{x\to\infty} \frac{-9x^5-x^2+x}{1+2x^3-3x^5}$

(c) [3] $\lim_{x\to 1^+} e^{-1/(\sqrt{x}-1)}$

(d) [3] $\lim_{x\to 0} \frac{\tan(3x)}{\sin(5x)}$

Question 4 [10 points]:

(a) [**3**] Find the general antiderivative: $f(x) = 4x^3 - 2e^x + \dfrac{1}{x}$

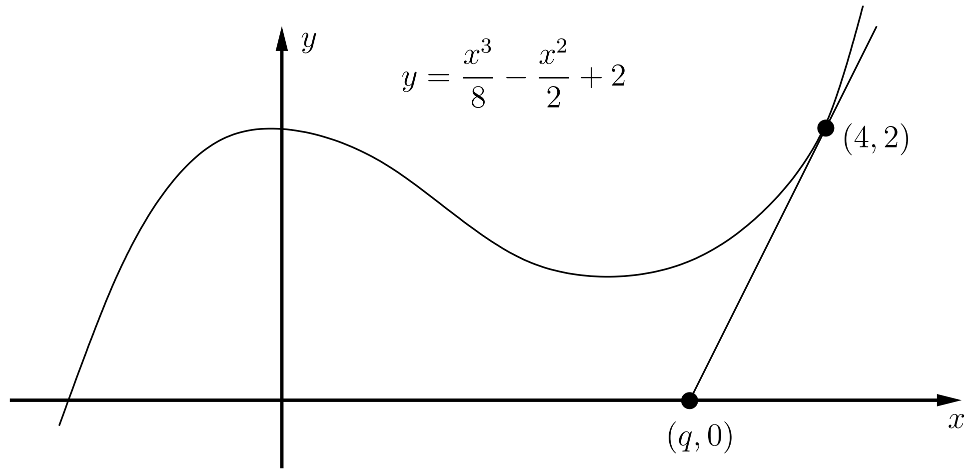
(b) [**3**] Find the general antiderivative: $f(x) = \dfrac{2x - \sqrt[3]{x^2} + 7}{x^3}$

(c) [**4**] A particle has acceleration $a(t) = 6t - 2$ where t is time in seconds. If the initial velocity is $v(0) = 3$ and initial position is $s(0) = 1$, determine the position of the particle at time $t = 2$ seconds.

Question 5 [8 points]: Let $f(x) = \dfrac{1}{\sqrt{x}}$. Use the definition of the derivative to find $f'(4)$. (No credit will be given if $f'(4)$ is found using differentiation rules.)

Question 6 [10 points]:

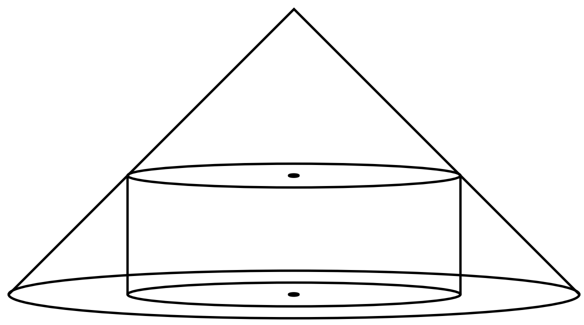
(a) [5] Determine the x-intercept q of the tangent line in the following figure.



(b) [5] Determine the linear approximation of $f(x) = x \sin(\pi x^2)$ at $a = 2$.

Question 7 [10 points]: A height of a cylinder is decreasing at 2 cm per minute. At what rate is the radius increasing at the instant when the height is 4 cm if the volume is a constant $V = \pi$ cm^3 at all times? State units with your answer. (Recall that the volume of a cylinder is $V = \pi r^2 h$.)

Question 8 [10 points]: A right circular cylinder is inscribed in a cone of height and base radius both equal to 3 cm. Find the largest possible volume of such a cylinder. Clearly justify all conclusions and state units with your answer. (Recall that the volume of a cylinder is $V = \pi r^2 h$, while that of a cone is $V = \pi r^2 h/3$.)



Question 9 [10 points]: A box with square base and no top is to have a volume of 6 m^3. Material for the bottom of the box costs $3 per square metre, while the material for the sides costs $2 per square metre. Determine the dimensions of the least expensive such box. Clearly justify all conclusions and state units with your answer.

Question 10 [10 points]:

(a) [5] Determine the equation of the tangent line to the curve

$$\sqrt{x+y} = 3 + x^2y^2$$

at the point $(0, 9)$. Implicit differentiation may help here.

(b) [5] Use logarithmic differentiation to find $\frac{dy}{dx}$:

$$y = \sqrt{x}e^{x^2}(x^2+1)^{10}$$

Question 11 [16 points]: For this question consider the function $f(x) = \dfrac{x^2 + 12}{x - 2}$.

(a) [1] Determine the domain of $f(x)$.

(b) [2] Determine the x and y intercepts of the graph of $y = f(x)$.

(c) [4] Determine the intervals of increase and decrease of $f(x)$.

(d) [1] State the local maxima and minima of $f(x)$.

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(e) [**2**] Using the fact that $f''(x) = \dfrac{32}{(x-2)^3}$ determine the intervals of concavity of $f(x)$ and identify inflection points, if any.

(f) [**2**] Determine the horizontal and vertical asymptotes, if any.

(g) [**4**] Use the information gathered in parts (a)-(f) to make an informative sketch of the graph of $y = f(x)$. Label your axes and any of the interesting points on your graph (intercepts, local extrema, etc.)