Method of Undetermined Coefficients

Use to solve

$$L(y) = f(x)$$

where

$$L = a_n D^n + a_{n-1} D^{n-1} + \cdots + a_1 D + a_0 \ .$$

First, determine the complementary function y_c which is the general solution to

$$L(y) = 0 \ .$$

Next, *guess* a particular solution y_p based on the form of $f(x)$. Consider y_p as a sum of trial functions determined as follows:

If $f(x)$ contains a term which is a constant multiple of ...	trial function to appear as a term of y_p*
k (a constant)	A
x^m	$c_m x^m + c_{m-1} x^{m-1} + \cdots + c_1 x + c_0$
e^{cx}	Ae^{cx}
$\sin(px+q)$	$A\sin(px+q) + B\cos(px+q)$
$\cos(px+q)$	$A\sin(px+q) + B\cos(px+q)$
$x^m e^{cx}$	$(c_m x^m + c_{m-1} x^{m-1} + \cdots + c_1 x + c_0)e^{cx}$
$x^m \sin(px+q)$	$(c_m x^m + c_{m-1} x^{m-1} + \cdots + c_1 x + c_0)\sin(px+q)$ $+(d_m x^m + d_{m-1} x^{m-1} + \cdots + d_1 x + d_0)\cos(px+q)$
$x^m \cos(px+q)$	$(c_m x^m + c_{m-1} x^{m-1} + \cdots + c_1 x + c_0)\sin(px+q)$ $+(d_m x^m + d_{m-1} x^{m-1} + \cdots + d_1 x + d_0)\cos(px+q)$
$e^{cx} x^m \sin(px+q)$	$e^{cx}(c_m x^m + c_{m-1} x^{m-1} + \cdots + c_1 x + c_0)\sin(px+q)$ $+e^{cx}(d_m x^m + d_{m-1} x^{m-1} + \cdots + d_1 x + d_0)\cos(px+q)$
$e^{cx} x^m \cos(px+q)$	$e^{cx}(c_m x^m + c_{m-1} x^{m-1} + \cdots + c_1 x + c_0)\sin(px+q)$ $+e^{cx}(d_m x^m + d_{m-1} x^{m-1} + \cdots + d_1 x + d_0)\cos(px+q)$

* If a constant multiple of a term of the trial function is already a term of the complementary solution y_c, multiply that trial function by x^j where j is the smallest natural number such that the new trial function no longer duplicates a term of y_c.