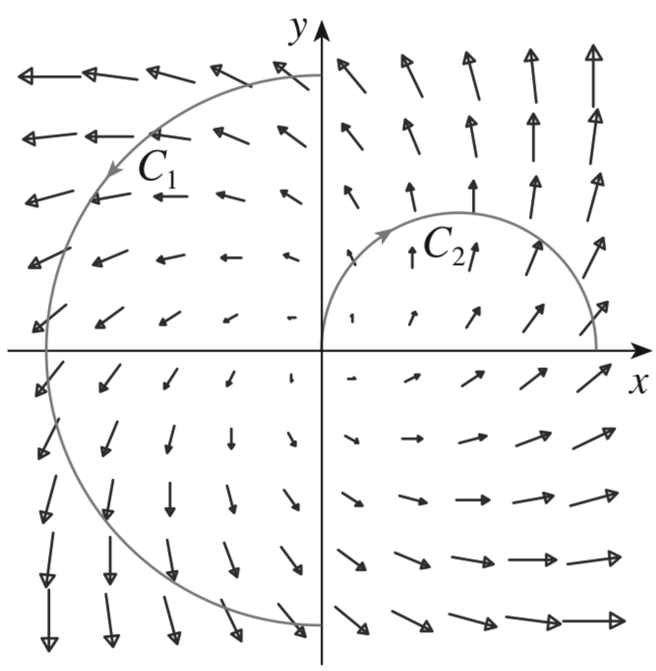
Question 1: At time $t = 1$ a particle is located at position $(1, 3)$ and moves according to the velocity field $\mathbf{F}(x, y) = \langle xy - 2, y^2 - 10 \rangle$. Estimate the particle's position at time $t = 1.05$.

[4]

Question 2: For this question use the following plot of the vector field $\mathbf{F}(x, y)$:



(a) Is $\int_{C_1} \mathbf{F} \cdot d\mathbf{r}$ positive, negative or zero? Give a brief explanation to support your answer.

[2]

(b) Is $\int_{C_2} \mathbf{F} \cdot d\mathbf{r}$ positive, negative or zero? Again, give a brief explanation to support your answer.

[2]

Question 3: Determine $\int_C xyz^2\, ds$ where C is the line segment from $(-1, 5, 0)$ to $(1, 6, 4)$.

[5]

Question 4: Determine the work done by the force field $\mathbf{F}(x, y) = \langle x^2, ye^x \rangle$ on a particle that moves along the parabola $x = y^2 + 1$ from $(1, 0)$ to $(2, 1)$.

[5]

Question 5: Let $\mathbf{F}(x, y, z) = \sin(y)\,\mathbf{i} + [x\cos(y) + \cos(z)]\,\mathbf{j} - y\sin(z)\,\mathbf{k}$.

(a) Find a potential function f for $\mathbf{F}$ (that is, a function f such that $\nabla f = \mathbf{F}$).

[5]

(b) Calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is given by $\mathbf{r}(t) = \langle \sin(t), t, 2t \rangle$, $0 \le t \le \pi/2$.

[3]

Question 6: Is $\mathbf{F}(x, y) = e^x \cos(y)\,\mathbf{i} + e^x \sin(y)\,\mathbf{j}$ conservative? Explain.

[2]

Question 7: Evaluate $\int_C \sqrt{1+x^3}\,dx + 2xy\,dy$ where C is the positively oriented triangle with vertices $(0,0)$, $(1,0)$ and $(1,3)$.

[7]

Question 8: Let $\mathbf{F} = \langle f(x), g(y) \rangle$ where f and g are differentiable and suppose C is a simple smooth positively oriented closed curve. Determine $\int_C \mathbf{F} \bullet d\mathbf{r}$.

[3]

Question 9: Let $\mathbf{F}(x, y, z) = e^{-x} \sin(y)\,\mathbf{i} + e^{-y} \sin(z)\,\mathbf{j} + e^{-z} \sin(x)\,\mathbf{k}$. Calculate

(a) div($\mathbf{F}$)

[2]

(b) curl($\mathbf{F}$)

[2]

Question 10: Is $\mathbf{F}(x, y, z) = xyz^2\,\mathbf{i} + x^2yz^2\,\mathbf{j} + x^2y^2z\,\mathbf{k}$ conservative? Explain.

[3]

Question 11: Let $\mathbf{F} = \langle xyz, -y^2z, yz^2 \rangle$. Could $\mathbf{F} = \text{curl}(\mathbf{G})$ for some vector field $\mathbf{G}$? Explain.

[3]