Strategy for Related Rate Problems

Related rate problems are problems involving relationships between quantities which are changing in time. Use a systematic approach to set up and solve them. Here are some guidelines, but keep in mind that every problem is different:

1. Thoroughly read and understand the problem. Play a little movie in your mind to visualize the situation. Do not start writing until you have a clear understanding of what is being described and what you are being asked to find. Identify statements describing changing quantities – these are statements about derivatives. Look for "action words", descriptions of quantities that are changing, growing, shrinking, increasing, decreasing, etc.

2. Sketch a diagram of the situation being described (if the problem lends itself to such a representation.)

3. Introduce variables and summarize the given information using "calculus statements". Identify quantities which are functions of time. Quantities that are increasing correspond to positive derivatives, decreasing quantities to negative derivatives. For example, if you are told that the volume of water in a vessel is decreasing by five litres per hour, you could say

   Let $V(t) =$ volume of water in vessel at time t

   $$\frac{dV}{dt} = -5 \text{ L/h}$$

4. Restate the problem as concisely as possible using calculus statements. Most related rate problems can be restated in the form

   Find $\boxed{}$ when $\boxed{} = \boxed{}$

   For example, if a problem asks you to determine how fast water level is increasing when the water volume is 10 L, the problem could be restated

   Find $\boxed{\dfrac{dh}{dt}}$ when $\boxed{V} = \boxed{10 \text{ L}}$

5. Find equations connecting known and unknown quantities. For example, a relationship between the volume of a cylinder and its height is

   $$V = \pi r^2 h$$

   where V, r, or h may be functions of time. Sometimes it may be necessary to eliminate variables in the equation using other other known relationships between variables.

6. Takes derivatives with respect to time of both sides of the equation found in the previous step.

7. After (<u>and only after!</u>) all derivative operations, insert the data from step 3 into the resulting equation and solve for the quantity of interest.

8. Write a concluding sentence and include appropriate units in your answer.

Example 1

The top of a ladder 5 m long rests against a vertical wall. The base of the ladder is pulled away from the wall at $1/3$ m/s. How fast is the top of the ladder slipping down the wall when it is 3 m above the base of the wall?

Example 2

An empty water tank in the shape of an inverted cone is 4 m deep and has a top diameter of 3 m. Water flows into the tank at a rate of 1 m^3/min. How fast is the water level rising when the depth of water in the tank is 2 m?

Example 3

An observer 2 km from the launch pad measures the angle of elevation of a launching rocket. When the angle is $\pi/6$ it is increasing by $1/16$ of a radian per second. Find the velocity of the rocket at that same instant.