Question 1: Expand and simplify: $(1-x+x^3)^2$

$= (1-x+x^3)(1-x+x^3)$

$= 1-x+x^3-x+x^2-x^4+x^3-x^4+x^6$

$= \boxed{x^6-2x^4+2x^3+x^2-2x+1}$

[See Algebra Review #16]

[3]

Question 2: Express as a single simplified fraction: $\frac{c}{ab^2}+\frac{a}{bc}+\frac{b}{ac}$

$= \boxed{\frac{c^2+a^2b+b^3}{ab^2c}}$

[See Algebra Review # 22]

[3]

Question 3: Factor completely: $8x^2+10x+3$

$= 8x^2+4x+6x+3$

$= 4x(2x+1)+3(2x+1)$

$= \boxed{(2x+1)(4x+3)}$

[See Algebra Review #36]

[3]

Question 4: Find all solutions: $x^3 - 2x + 1 = 0$

[See Algebra Review #67.]

$x=1$ is a solution, so $x-1$ is a factor of x^3-2x+1:

$$\begin{array}{r} x^2 + x - 1 \\ x-1 \overline{) x^3+0x^2-2x+1} \\ -(x^3-x^2) \\ \hline x^2-2x+1 \\ -(x^2-x) \\ \hline -x+1 \\ -(-x+1) \\ \hline 0 \end{array}$$

$(x-1)(x^2+x-1)=0$

So $x=1$ or $\underbrace{x^2+x-1=0}$

$$x = \frac{-1 \pm \sqrt{1^2-4(1)(-1)}}{2(1)} = \frac{-1\pm\sqrt{5}}{2}$$

so solutions are

$$\boxed{x=1,\ \frac{-1+\sqrt{5}}{2},\ \frac{-1-\sqrt{5}}{2}}$$

[4]

Question 5: Simplify and express your answer using only positive exponents: $\left(\frac{3x^{3/2}y^3}{x^2y^{-1/2}}\right)^{-2}$

[See Algebra Review #93, 94, 99]

$$= \frac{3^{-2}x^{-3}y^{-6}}{x^{-4}y}$$

$$= \boxed{\frac{x}{9y^7}}$$

[3]

Question 6: Find an equation of the line that passes through the midpoint of $A(-7,4)$ and $B(5,-12)$ and which is perpendicular to the line through these two points.

midpoint is $C = \left(\frac{-7+5}{2}, \frac{4+(-12)}{2}\right) = (-1,-4)$

Line through AB has slope $m_{AB} = \frac{-12-4}{5-(-7)} = \frac{-16}{12} = \frac{-4}{3}$.

Slope of our line is then $m = \frac{-1}{m_{AB}} = \frac{3}{4}$

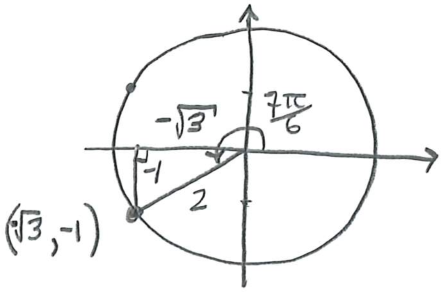
Using $m=\frac{3}{4}$ and point $C(-1,-4)$:

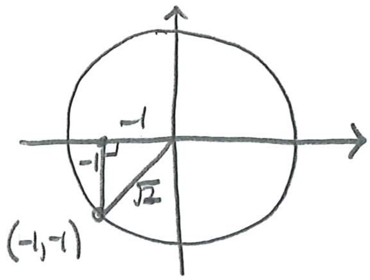
$$\boxed{y+4 = \frac{3}{4}(x+1)}$$

[See Analytic Geometry Review #45.]

[4]

Question 7: Determine $\sin(7\pi/6) - \sec(5\pi/4)$. Express your answer as a single simplified fraction.





⌈See Appendix A #23, 25, 27⌋

$$\sin\left(\frac{7\pi}{6}\right) - \sec(5\pi/4)$$

$$= \frac{-1}{2} - \frac{(-\sqrt{2})}{1} = \boxed{\frac{2\sqrt{2}-1}{2}}$$

[3]

⌈See Appendix A #65, 67, 61.⌋

Question 8: Find all values of x in the interval $[0, 2\pi]$ for which $2\sin(x) = \tan(x)$.

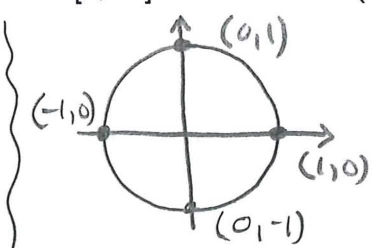
$$2\sin(x) = \tan(x)$$

$$2\sin(x) = \frac{\sin(x)}{\cos(x)}$$

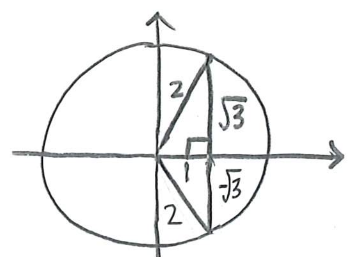
$$2\sin(x)\cos(x) = \sin(x)$$

$$\sin(x)\left[2\cos(x) - 1\right] = 0$$

$$\therefore \sin(x) = 0, \quad \cos(x) = \frac{1}{2}$$



$\sin(x) = 0$ for $x = 0, \pi, 2\pi$



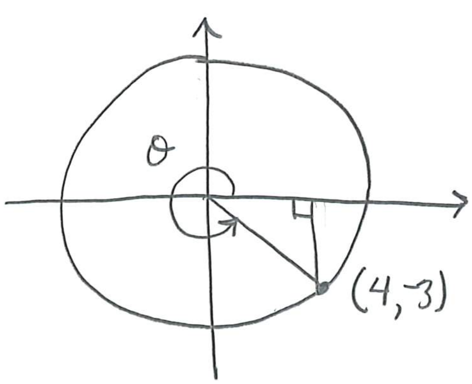
$\cos(x) = \frac{1}{2}$ for $x = \frac{\pi}{3}, 5\pi/3$.

so solutions are

$$\boxed{x = 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}, 2\pi}$$

[4]

Question 9: If $\tan(\theta) = -3/4$ where $\frac{3\pi}{2} < \theta < 2\pi$ then determine $\csc(\theta)$.



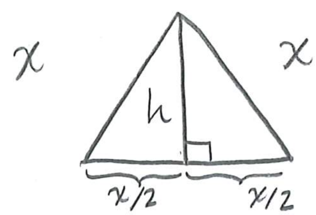
$$r = \sqrt{4^2 + (-3)^2} = 5$$

$$\therefore \csc(\theta) = \frac{r}{y} = \boxed{\frac{-5}{3}}$$

⌈See Appendix A #29, 31, 33.⌋

[3]

Question 10: Express the area A of an equilateral triangle (that is, a triangle having all sides of equal length) as a function of the length x of one of its sides.



$$h=\sqrt{x^2-\left(\tfrac{x}{2}\right)^2}=\frac{\sqrt{3}}{2}\,x$$

See 1.1 #49

$$A=\frac{1}{2}xh$$

$$A=\frac{1}{2}(x)\left(\frac{\sqrt{3}}{2}x\right)$$

$$A=\boxed{\frac{\sqrt{3}x^2}{4}}$$

[4]

Question 11: Evaluate and simplify the difference quotient $\dfrac{f(x+h)-f(x)}{h}$ where $f(x)=\dfrac{5}{x^2}$. Express your final answer as a single simplified fraction.

$$\frac{f(x+h)-f(x)}{h}=\frac{\left[\frac{5}{(x+h)^2}-\frac{5}{x^2}\right]}{h}$$

$$=\frac{1}{h}\left[\frac{5x^2-5(x+h)^2}{(x+h)^2x^2}\right]$$

$$=\frac{1}{\cancel{h}}\left[\frac{\cancel{5x^2}-\cancel{5x^2}-10\cancel{h}x-5h^{\cancel{2}}}{(x+h)^2x^2}\right]$$

$$=\boxed{\frac{-5(2x+h)}{(x+h)^2x^2}}$$

See 1.1 #21,23

[4]

Question 12: Evaluate the limits:

(i) $\displaystyle\lim_{x\to 4}\frac{x^2-6x+5}{x^2-4}=\frac{-3}{12}=\boxed{\frac{-1}{4}}$

[1]

(ii) $\displaystyle\lim_{x\to -2}\frac{\sqrt{x^2+5}}{x^2-4}$ $\;\to$ "$\frac{3}{0}$" so $\boxed{\text{limit does not exist}}$

[1]

Question 13: Evaluate the following limits, if they exist:

(a) $\lim_{x\to 0} \frac{x}{\sqrt{4+x}-\sqrt{4-x}}$ $\sim$ "$\frac{0}{0}$"

$= \lim_{x\to 0} \frac{x}{\sqrt{4+x}-\sqrt{4-x}} \cdot \frac{\sqrt{4+x}+\sqrt{4-x}}{\sqrt{4+x}+\sqrt{4-x}}$

$= \lim_{x\to 0} \frac{x(\sqrt{4+x}+\sqrt{4-x})}{4+x-(4-x)}$

$= \lim_{x\to 0} \frac{\cancel{x}(\sqrt{4+x}+\sqrt{4-x})}{2\cancel{x}} = \frac{4}{2} = \boxed{2}$

[3]

(b) $\lim_{x\to 3} \frac{x^2-6x+9}{x^2-9}$ $\sim$ "$\frac{0}{0}$"

$= \lim_{x\to 3} \frac{\cancel{(x-3)}(x-3)}{\cancel{(x-3)}(x+3)}$

$= \boxed{0}$

[3]

(c) $\lim_{x\to 2} \left(\frac{1}{x-2} - \frac{4}{x^2-4}\right)$ $\sim$ "$\frac{1}{0} - \frac{4}{0}$"

$= \lim_{x\to 2} \left(\frac{1}{x-2} - \frac{4}{(x-2)(x+2)}\right)$

$= \lim_{x\to 2} \frac{x+2-4}{(x-2)(x+2)}$

$= \lim_{x\to 2} \frac{\cancel{(x-2)}}{\cancel{(x-2)}(x+2)} = \boxed{\frac{1}{4}}$

[4]