

Question 1:

(i) Simplify: $\operatorname{Im}(e^{\cos(i\pi)})$

[2]

(ii) Using the principal value, express in the form $a + ib$ where a and b are real: $(1 + i)^{1/2}$

[3]

Question 2: Find all solutions to

(i) $e^z = i\pi$

[3]

(ii) $\operatorname{Log}(1 + z) = \frac{3\pi i}{2}$

[2]

Question 3: Find all solutions to $\sin(z) = \frac{\sqrt{2}}{2}$.

[5]

Question 4: Let $f(z) = z^z$ be defined using the principal value of the logarithm. Compute $f'(1)$.

[5]

Question 5: Calculate $\int_{\gamma} \operatorname{Im}(z^2) dz$ where $\gamma(t) = t + \frac{i}{t}, 1 \leq t \leq 2$.

[5]

Question 6: Calculate $\int_{\gamma} e^z \cos(e^z) dz$ where γ is the right hand side of the circle $|z| = \pi$ from $-i\pi$ to $i\pi$.

[5]

Question 7: Evaluate $\int_{C[i,2]} \frac{\cos(z)}{z(z-3)} dz$ where the path $C[i, 2]$ has positive orientation.

[5]

Question 8: Evaluate $\int_{C[i,5]} \frac{\cos(z)}{z(z-3)} dz$ where the path $C[i, 5]$ has positive orientation.

[5]
