Question 1: Simplify and express your answer in the form $a + ib$ where a and b are real:

$$\frac{2+3i}{1+2i} + \frac{8+i}{2-i}$$

[5]

Question 2: Express in form $z = re^{i\theta}$ where r and θ are real:

$$\frac{\sqrt{2}(1+i)}{\sqrt{3}+i}$$

[5]

Question 3: Determine all values of $\left(\frac{2i}{1+i}\right)^{1/2}$.

[5]

Question 4: Find all pairs (a, b) so that $f(x + iy) = ax^2 + iabxy + by^2$ is entire.

[5]

Question 5:

(i) Find the points, if any, at which $f(z) = (\overline{z+i})^2$ is differentiable. For each such point explain why the function is differentiable and state the value of the derivative.

[4]

(ii) Find the points, if any, at which $f(z) = (\overline{z+i})^2$ is holomorphic. Explain.

[2]

Question 6: Suppose f is holomorphic in a region D and $f(z) \in \mathbb{R}$ for every $z \in D$. Explain why f must be a constant function. (Hint: consider $f = u + iv$.)

[4]

Question 7: Sketch the image of S under the mapping $f(z) = \dfrac{-1}{\overline{z}}$ where

$$S = \{z \in \mathbb{C} \mid 1 \le |z| \le 2 \text{ and } 0 \le \arg(z) \le \pi/2\}$$

[5]

Question 8: Determine the absolute maximum value of $\left|\dfrac{1+z^2}{z^2}\right|$ on the closed disk $S = \{z \in \mathbb{C} \,:\, |z+2| \le 1\}$.

[5]