The final exam will be comprehensive, covering all material since the beginning of the course. In addition to reviewing your assignments and class notes, I suggest you work through the list of sample exam questions below.

Cheat Sheet and Calculator

As with tests, a single double-sided letter-size handwritten "cheat sheet" containing formulae, theory and numerical values may be used for the exam. The cheat sheet may not contain worked examples however, and must be submitted when you hand in your test.

A standard non-graphing scientific calculator may be used.

Sample Questions

1. Simplify and express in the form $a+bi$: $\dfrac{5+5i}{(1+3i)(\frac{1}{2}-\frac{i}{2})}$.
2. Determine and sketch all cube roots of $8(1-\sqrt{3}i)$.
3. Evaluate $|e^{iz}|$ if $z = 6e^{i\pi/3}$.
4. Determine the points, if any, at which $f(z) = |\bar{z} - i|^2$ is holomorphic.
5. Determine the harmonic conjugate of $u(x,y) = 3x^2y - y^3 + x + 4xy$.
6. Can $u(x,y) = xy^2$ be the real part of an entire function? Explain.
7. Find all values of $[\mathrm{Log}(i)]^{\mathrm{Log}(i)}$.
8. Find all solutions to $e^{iz} = i$.
9. Evaluate $\displaystyle\int_\gamma \mathrm{Re}(z)\,dz$ where
   (a) γ is a line from $z = 0$ to $z = 1+i$
   (b) γ is a line segment from from $z = 0$ to $z = i$ followed by a line segment from $z = i$ to $z = 1+i$.
10. Evaluate $\displaystyle\int_C \frac{\cos z}{e^z - 1}\,dz$ where C is the circle $|z - 2i| = 1$ traversed once in the positive direction.
11. Evaluate $\displaystyle\int_\gamma \frac{1}{z}\,dz$ where γ is any simple path from $z = -2$ to $z = -i$ which does not leave the third quadrant.
12. Evaluate $\displaystyle\int_C \frac{z^3}{(z+i)(z+2)^2}\,dz$ where C is the circle

(a) $|z| = 1/2$

(b) $|z| = 3/2$

(c) $|z + 2| = 1/2$

(d) $|z| = 3$

In each case the circle is traversed once in the positive direction.

13. Show that $f(z) = \dfrac{1 + \cos(\pi z)}{(z^2 - 1)^2}$ has a removable singularity at $z = -1$ (You may use L'Hospital's rule here.)

14. Classify the isolated singularities of the following as either removable, a pole, or essential:

(a) $\dfrac{1}{e^z - 1}$

(b) $\cos\left(1 - \dfrac{1}{z}\right)$

15. Use the residue theorem to evaluate the following integrals. In each case the circles are traversed once in the positive direction:

(a) $\displaystyle\int_{|z|=2} \frac{z^3 + 2z}{z - i}\, dz$

(b) $\displaystyle\int_{|z|=1} z^2 e^{1/z}\, dz$

16. Determine $\displaystyle\int_{-\infty}^{\infty} \frac{x}{(x^2 + 4x + 13)^2}$ using the Residue theorem.