1. Read through example 5.14 in the textbook and use the same method to calculate the value of the real integral $\displaystyle\int_{-\infty}^{\infty} \frac{1}{1+x^4}\,dx$.

2. In this exercise we'll prove one form of the *Maximum Modulus Principle*: Suppose $f(z)$ is holomorphic on the region G containing the simple closed positively oriented piecewise smooth contractible path γ and let S be the set of points inside and on γ (so S is closed and includes γ as its boundary.) Then the absolute maximum of $|f(z)|$ on S occurs on the boundary γ itself.

   Proceed as follows: Let z_0 be any point strictly inside γ and let M be the maximum of $|f(z)|$ on γ. We will show that $|f(z_0)| \leq M$.

   (a) Let $n \geq 1$ be an integer. Then

   $$[f(z_0)]^n = \frac{1}{2\pi i}\int_\gamma \frac{[f(\zeta)]^n}{\zeta - z_0}d\zeta \quad \text{(why?)} \tag{1}$$

   (b) Let μ be the minimum distance from z_0 to γ and $\ell(\gamma)$ be the length of γ. Use (1) to show that

   $$|f(z_0)|^n \leq \frac{1}{2\pi}\frac{M^n}{\mu}\ell(\gamma) \tag{2}$$

   (c) Take (real) n^{th} roots of both sides of (2) and then let $n \to \infty$.

3. The power series $f(z) = \displaystyle\sum_{n=1}^{\infty} \frac{(z+1)^n}{(n+5)^3 3^n}$ defines a holomorphic function on the largest open disk $D[z_0, R]$ on which it converges. Determine the center and radius of this disk.

4. For each of the following, determine the largest open disk on which the Taylor series converges

   (a) $\dfrac{\sin z}{z^2+4}$ about $z = 0$.

   (b) $\dfrac{e^z}{z^2 - z}$ about $z = 4i$.

5. Determine the Taylor series for $f(z) = \sinh(z)\cosh(z)$ about $z = 0$ and state the radius of convergence.

6. Find a Laurent series expansion for $f(z) = \dfrac{e^{(z^2)}}{z^3}$ about $z = 0$.