

NOTE: Some of the exercises require that you sketch a set. When doing so, clearly indicate which points are in the set and which points are not. For example, boundary lines or curves which are not in the set should be indicated with dashes or dots.

1. (a) Show that $v(x, y) = \frac{x}{x^2 + y^2}$ is harmonic on $D = \mathbb{C} \setminus \{0\}$.
(b) Find a function $f(z) = u(x, y) + iv(x, y)$ that is analytic on D .
(c) Express $f(z)$ in (b) in terms of the variable z .
2. (a) Calculate $\tan(\pi - 2i)$. You should be able to state your final answer in a tidy form using hyperbolic trigonometric functions.
(b) Find all solutions to $\cos(z) = i \sin(z)$
(c) Find all solutions to $\cos(z) = \sin(z)$
3. Compute the following logarithms:
(a) $\text{Log}(-e^2)$
(b) $\text{Log}(-12 + 5i)$
(c) $\text{Log}[(1 + \sqrt{3}i)^5]$
4. Find the image of the given set under the mapping $w = \text{Log}(z)$:
(a) The circle $|z| = 4$
(b) the region in the first quadrant bounded between the circles $|z| = 1$ and $|z| = e$
5. Find the principal value of the given complex power:
(a) $i^{i/\pi}$
(b) $(1 + i)^{2-i}$
6. For $\alpha, \beta \in \mathbb{C}$, is it true in general (using principal values) that $(z^\alpha)^\beta = z^{\alpha\beta}$?
(Consider $[(i - 1)^2]^i$.)
7. For each of the following, determine the largest subset of \mathbb{C} on which the function is holomorphic (use principal values here):
(a) $(z - 3)^i$
(b) i^{z-3}
8. Find all solutions to $\cos(z) = 2$.