

NOTE: Some of the exercises require that you sketch a set. When doing so, clearly indicate which points are in the set and which points are not. For example, boundary lines or curves which are not in the set should be indicated with dashes or dots.

1. Show that if  $f'(z_0)$  exists then  $f$  is continuous at  $z_0$ .
2. Use Proposition 2.10 (Properties of Derivatives) to prove that polynomial functions are entire.
3. Textbook exercise 2.18 parts (b), (h), (i) and (l)
4. Express each of the following functions in the form  $w = u(x, y) + iv(x, y)$  where  $u$  and  $v$  are real:

(a)  $f(z) = \frac{\bar{z}}{z+1}$

(b)  $f(z) = z + \frac{1}{z}$

(c)  $f(z) = e^{2z+i}$

(d)  $f(z) = e^{z^2}$

5. For each of the following, sketch the set  $S$  and then sketch the image of  $S$  under the given function:

(a)  $S = \{z \in \mathbb{C} \mid \operatorname{Re}(z) > 0 \text{ and } \operatorname{Im}(z) > 0\}$ ,  $f(z) = -3iz$

(b)  $S = \{z \in \mathbb{C} \mid \operatorname{Re}(z) > 1\}$ ,  $f(z) = (z+2)^2$

(c)  $S = \left\{z \in \mathbb{C} \mid \frac{\pi}{4} < \operatorname{Im}(z) < \frac{\pi}{2}\right\}$ ,  $f(z) = e^z$

6. Prove that  $\overline{(e^z)} = e^{\bar{z}}$ .

7. For each of the following determine the set of points on which the function is differentiable:

(a)  $f(z) = e^{x^2-y^2} \cos(2xy) + ie^{x^2-y^2} \sin(2xy)$

(b)  $f(z) = \frac{x^3 + xy^2 + x}{x^2 + y^2} + i \frac{x^2y + y^3 - y}{x^2 + y^2}$

8. Find real constants  $a$ ,  $b$ ,  $c$  and  $d$  so that  $f(z) = x^2 + axy + by^2 + i(cx^2 + dxy + y^2)$  is entire.
9. The function  $f(z) = x^2 - x + y + i(y^2 - 5y - x)$  is not holomorphic at any point, but is differentiable along a line in the complex plane. Find the line.
10. Textbook exercise 3.32.