

NOTE: Some of the exercises require that you sketch a set. When doing so, clearly indicate which points are in the set and which points are not. For example, boundary lines or curves which are not in the set should be indicated with dashes or dots.

1. Find all values of  $(1 + i\sqrt{3})^{3/4}$
2. Let  $n$  be a nonnegative integer. Determine (with justification) all values of  $n$  such that  $z^n = 1$  possesses only real solutions.
3. Suppose  $w$  is located in the first quadrant and is a cube root of a complex number  $z$ . Can there exist a second cube root of  $z$  also located in the first quadrant? Justify your answer.
4. For each of the following, sketch the set  $S$  of points in the complex plane satisfying the inequality and state whether the set is (i) open, (ii) a region, (iii) bounded, or (iv) connected:
  - (a)  $S = \{z = re^{i\theta} \in \mathbb{C} : |\theta| \leq 5\pi/6\}$
  - (b)  $S = \{z \in \mathbb{C} : 2 < \operatorname{Re}(z - 1) < 4\}$
  - (c)  $S = \{z \in \mathbb{C} : \operatorname{Re}(z^2) > 0\}$
  - (d)  $S = \{z \in \mathbb{C} : 2 \leq |z - 3 + 4i| \leq 5\}$
5. Let  $S$  be the set consisting of the complex plane with the circle  $|z| = 3$  removed. Determine the boundary points of  $S$ . Is  $S$  connected? Explain.
6. Show that  $f(z) = |z|$  is continuous on  $\mathbb{C}$ .
7. Find the following limits (if the limit does not exist, say so):
  - (a)  $\lim_{z \rightarrow i} 3z^2 + 7iz - 2 - i$
  - (b)  $\lim_{z \rightarrow i} \frac{z^4 + 1}{z - i}$
  - (c)  $\lim_{z \rightarrow i} \frac{z^4 - 1}{z - i}$
  - (d)  $\lim_{z \rightarrow 1+i} \frac{z^2 + z - 1 - 3i}{z^2 - 2z + 2}$
8. Give an argument which shows that  $\lim_{z \rightarrow 0} \frac{\operatorname{Re}(z)}{\operatorname{Im}(z)}$  does not exist.