

1. Express each of the following complex numbers in the form  $a + ib$  with  $a$  and  $b$  real:

(a)  $\frac{(8 + 2i) - (1 - i)}{(2 + i)^2}$

(b)  $(2 + i)(-1 - i)(3 - 2i)$

2. Let  $z$  be a complex number with  $\text{Im}(z) > 0$ . Prove that  $\text{Im}(1/z) < 0$ .

3. Let  $z = 3 - 2i$ . Plot the points  $z$ ,  $-z$ ,  $\bar{z}$ ,  $-\bar{z}$  and  $1/z$  in the complex plane.

4. Sketch and describe the set of points in the complex plane that satisfies each of the following:

(a)  $|2z - i| = 4$

(b)  $|z| = \text{Re}(z) + 2$

(c)  $\text{Re}(z) \geq 4$

5. Prove that if  $(\bar{z})^2 = z^2$  then  $z$  is either pure real or pure imaginary.

6. Let  $z = 2 - i$  and  $w = 1 + i$ . Sketch each of the following vectors.

(a)  $z + w$

(b)  $z - w$

(c)  $2z - 3w$

7. Find  $\arg(z)$  for each of the following where  $0 \leq \arg(z) < 2\pi$

(a)  $z = -6 - 6i$

(b)  $z = \sqrt{3} - i$

8. Express each of the following in the form  $a + bi$  where  $a, b \in \mathbb{R}$ :

(a)  $\frac{e^{3i} - e^{-3i}}{2i}$

(b)  $2e^{3+i\pi/6}$

9. Express each of the following complex numbers in exponential form  $re^{i\theta}$  with  $r > 0$  and  $0 \leq \theta < 2\pi$ :

(a)  $(\cos(2\pi/9) + i \sin(2\pi/9))^3$

(b)  $\frac{2 + 2i}{-\sqrt{3} + i}$

10. Show that  $|e^z| \leq 1$  for all complex numbers  $z$  with  $\text{Re}(z) \leq 0$ .